- This exam is worth 150 points and has 7 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on both sides.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/121824 (24 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) The function $f(x, y) = \sqrt{x^2 + y^2}$ has no critical points.

   (b) For any smooth path, $\mathbf{r}(t)$, in $\mathbb{R}^3$, the volume of the parallelepiped formed by $\mathbf{T}, \mathbf{N}, \mathbf{B}$ is 1.

   (c) The first degree Taylor polynomial centered at the origin of any function of the form $f(x, y) = ax + by + c$ is $f(x, y)$.

   (d) The graph of the equation $\rho^2 - 2\rho \sin\phi \cos\theta = 0$ is a sphere of radius 1 centered at $(0, 1, 0)$.

   (e) The line with symmetric equations $x - 4 = \dfrac{y+1}{-2} = z - 3$ never intersects the plane $x + y + z = 1$.

   (f) Suppose that for a given function $f_x(x, y) = (x-1)^2 - y$ and $f_y(x, y) = y - x + 1$. Then $f(x, y)$ has a local maximum at $(2, 1)$.

   (g) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2}{x^2 + y^3}$ does not exist.

   (h) The vertical traces of $z - y - x^2 = 0$ in planes parallel to the yz-plane are lines.

2. [2350/121824 (15 pts)] The area of a triangle is $A(a, b, C) = \dfrac{1}{2}ab\sin C$ where a and b are two sides of the triangle and C is the angle between the sides of length a and b. In surveying a particular triangular plot of land, the sides a and b are measured to be 120 and 200 feet, respectively, and C is read to be $\pi/3$ radians. By how much is the computed area in error if the sizes of each side are in error by 0.1 foot and C is in error by $\pi/60$ radians?

3. [2350/121824 (17 pts)] Ralphie the buffalo is part of a herd on the eastern plains of Colorado containing $\delta(x, y) = 60y$ buffalo per square kilometer. A ranch on the plains is given by the region, $\mathcal{D}$, bounded by the x- and y-axes and the line $x + y = 1$ where distances are measured in kilometers.

   (a) (5 pts) Fully set up, but **do not evaluate**, an integral that computes the number of buffalo on the ranch.

   (b) (12 pts) Ralphie decides to lead a stampede of the buffaloes to Folsom Field with the velocity vector field of the herd given by

   $$\mathbf{V} = \left(2x - y^4\right)\mathbf{i} + x^2\,\mathbf{j} \;\text{ km/hour}$$

   The flux vector field, $\mathbf{F} = \delta\mathbf{V}$, gives the number of buffaloes per hour per kilometer and can be used to determine the flux (buffaloes per hour) of the herd passing across a curve in the plane. Use one of the Fundamental Theorems that we studied this semester to find the net number of buffaloes per hour exiting across the boundary of the ranch (region $\mathcal{D}$).

MORE PROBLEMS BELOW/ON REVERSE

4. [2350/121824 (18 pts)] Some friends of yours are running laps in a counterclockwise direction around the unit circle in the presence of the force field $\mathbf{F} = \left(Axy - y^3\right)\mathbf{i} + \left(4y + 3x^2 - 3xy^2\right)\mathbf{j}$ where A is a constant.

   (a) (6 pts) Having gone from $(1, 0)$ to $(0, 1)$, they are complaining about how much work they have done. You tell them to stop whining because when they get back to $(1, 0)$ they will have done no work at all. Indeed, they could walk in any closed path and not do any work. What is A?

   (b) (12 pts) To get their minds off all the work they are doing, you ask them to walk along the path, $\mathcal{C}$, parameterized by

   $$\mathbf{r}(t) = \left(2e^{t^2-t} + t\right)\mathbf{i} + t\cos(\pi t)\,\mathbf{j},\ 0 \le t \le 1$$

   instead. Find the work done by the force field on your friends during that adventure.

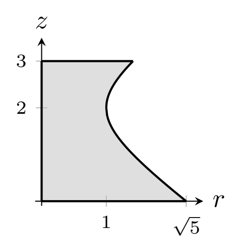
5. [2350/121824 (31 pts)] A cooling tower is in the shape of $(z-2)^2 = x^2 + y^2 - 1,\ 0 \le z \le 3$. The vector field describing the movement of heat is $\mathbf{H} = \left\langle 2xz + y^2, e^{xz}, z^2 \right\rangle$. Your mission is to compute the heat flux through the top, bottom and side of the tower, a closed surface.

   (a) (3 pts) Name the surface.

   (b) (4 pts) Without evaluating any integrals, describe in words why $\displaystyle\iint_{\mathcal{S}_b} \mathbf{H}\cdot\mathbf{n}\,\mathrm{d}S = 0$, where $\mathcal{S}_b$ is the portion of the tower in the xy-plane.

   (c) (12 pts) By direct calculation, find the upward heat flux through the top, $\mathcal{S}_t$, of the tower.

   (d) (12 pts) With the aid of one of the Fundamental Theorems that we have studied, find the heat flux through the side, $\mathcal{S}$, of the tower. The following diagram shows a cross section (constant θ) through the tower.



6. [2350/121824 (27 pts)] Let $\mathbf{F} = y\,\mathbf{i} - x\,\mathbf{j} - z^2\,\mathbf{k}$.

   (a) (12 pts) Directly evaluate $\displaystyle\iint_{\mathcal{S}} \nabla\times\mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S$, where $\mathcal{S}$ is the portion of the sphere of radius 2 centered at the origin below the plane $z = -1$ oriented with an upward pointing normal.

   (b) (12 pts) Directly evaluate $\displaystyle\int_{\mathcal{C}} y\,\mathrm{d}x - x\,\mathrm{d}y - z^2\,\mathrm{d}z$, where $\mathcal{C}$ is the circle of radius $\sqrt{3}$ centered at the origin in the plane $z = -1$ oriented counterclockwise when viewed from above.

   (c) (3 pts) Why are the answers to parts (a) and (b) the same?

7. [2350/121824 (18 pts)] You have just purchased a piece of land on which to build a ski resort. The piece of land can be described as the region in the xy-plane satisfying the inequalities $|x| \le 10$ and $|y| \le 9$. The elevation of the terrain in the resort is

   $$f(x, y) = x - \frac{1}{12}x^3 - \frac{1}{4}y^2 + 6$$

   (a) (4 pts) You want to place some weather instruments on the highest point and a restaurant at the lowest point in the ski resort. **Without** doing any calculations, can this be accomplished? Why or why not?

   (b) (7 pts) The midpoint of one of the ski runs will be above the point $(x, y) = (4, 2)$. Find a unit vector giving the direction that will produce the "most downhill" (*i.e.* "most negative", minimum) slope of the run at that point. What will the minimum slope be?

   (c) (7 pts) Suppose you are cross-country skiing on the path whose projection on the xy-plane is $\mathbf{r}(t) = t\,\mathbf{i} + \dfrac{t^2}{2}\mathbf{j},\ -4 \le t \le 4$. Use an appropriate Calculus 3 chain rule to determine your rate of change of elevation when you are at the point $\left(1, \frac{1}{2}, \frac{329}{48}\right)$.