

ASEN 6519

VERIFICATION & CONTROL SYNTHESIS FOR STOCHASTIC SYSTEMS

SPRING 2023

LECTURE INFORMATION

Tuesdays and Thursdays: 11:30 am - 12:45 pm

Room: AERO N240

Video recording will be made available after each lecture on the course canvas page

INSTRUCTOR

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Office hour: Thursdays 2-3pm and by appointment

COURSE DESCRIPTION

Real-world phenomena and processes by and large include randomness in their nature. To mathematically describe them, stochastic models provide a fundamental tool with major applications in physics, biology, finance, and engineering. This course introduces formal verification and control synthesis for stochastic systems with emphasis on safety-critical applications.

The course consists of two parts. In the first part, the focus is on systems with discrete state spaces. We start by introducing a modeling framework for such stochastic systems, e.g., discrete- and continuous-time Markov chains and Markov decision processes. We then introduce appropriate formal languages to specify behaviors of such systems. Next, we introduce a set of techniques that can be used for design and analysis of these stochastic models. In the second part of the course, we focus on continuous-space stochastic systems. We introduce verification and control synthesis techniques for these systems by means of discrete abstractions.

This course is designed to be aligned with the objectives of the CEAS's Autonomous Systems Interdisciplinary Research Theme and is open to AES, CS, ME, and ECEE students.

PREREQUISITES

The course is essentially self-contained, and students are only expected to be familiar with linear algebra, basics of probabilities and difference equations, and graph theory.

Recommended preparation: students should be proficient in linear algebra, basic notions of probabilities and difference equations, and some discrete math (graphs and transition systems). Basic knowledge of controls concepts is helpful but not essential.

GRADING AND EVALUATION

Classwork consists of:

- homework 20%
- tool exercises 10%
- mid-term exam 30%
- paper presentation 10%
- final project 30%

TEXTBOOKS

Required:

- *Principles of Model Checking*
Christel Baier and Joost-Pieter Katoen
MIT Press
2008
e-book through CU library: <https://tinyurl.com/yxoxgjav>

Additional Resources:

- *Stochastic Model Checking*
M. Kwiatkowska, G. Norman, and D. Parker
Springer
2007
Link: <https://www.prismmodelchecker.org/papers/sfm07.pdf>
- *Automated Verification Techniques for Probabilistic Systems*
V. Forejt, M. Kwiatkowska, G. Norman and D. Parker
Springer
2011
Link: <https://www.prismmodelchecker.org/papers/sfm11.pdf>

- *Temporal logic motion planning and control with probabilistic satisfaction guarantees*
M. Lahijanian, S. B. Andersson, and C. Belta
IEEE Transactions on Robotics
2012
Link: <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6085615>
- *Formal Verification and Synthesis for Discrete-Time Stochastic Systems*
M. Lahijanian, S. B. Andersson, and C. Belta
IEEE Transactions on Automatic Control
2015
Link:
<https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7029024&isnumber=7166373>

COURSE OUTLINE

- Discrete-time Markov chains (DTMCs) and their properties
- Probabilistic temporal logics: PCTL, LTL, etc.
- PCTL model checking for DTMCs
- The PRISM model checker
- Costs & rewards
- Continuous-time Markov chains (CTMCs)
- Applications (CTMC)
- Markov decision processes (MDPs)
- Strategy synthesis
- Probabilistic LTL model checking
- Applications (MDP)
- Discrete-time, continuous stochastic difference equations
- MDP approximate abstraction
- Multi-objective analysis

CLASSROOM BEHAVIOR

Both students and faculty are responsible for maintaining an appropriate learning environment in all instructional settings, whether in person, remote or online. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, veteran status, political affiliation or political philosophy. For more information, see

the [classroom behavior](#) policy, the [Student Code of Conduct](#), and the [Office of Institutional Equity and Compliance](#).

REQUIREMENTS FOR COVID-19

As a matter of public health and safety, all members of the CU Boulder community and all visitors to campus must follow university, department and building requirements and all public health orders in place to reduce the risk of spreading infectious disease. CU Boulder currently requires COVID-19 vaccination and boosters for all faculty, staff and students. Students, faculty and staff must upload proof of vaccination and boosters or file for an exemption based on medical, ethical or moral grounds through the MyCUHealth portal.

The CU Boulder campus is currently mask-optional. However, if public health conditions change and masks are again required in classrooms, students who fail to adhere to masking requirements will be asked to leave class, and students who do not leave class when asked or who refuse to comply with these requirements will be referred to Student Conduct and Conflict Resolution. For more information, see the policy on classroom behavior and the Student Code of Conduct. If you require accommodation because a disability prevents you from fulfilling these safety measures, please follow the steps in the “Accommodation for Disabilities” statement on this syllabus.

If you feel ill and think you might have COVID-19, if you have tested positive for COVID-19, or if you are unvaccinated or partially vaccinated and have been in close contact with someone who has COVID-19, you should stay home and follow the further guidance of the Public Health Office (contacttracing@colorado.edu). If you are fully vaccinated and have been in close contact with someone who has COVID-19, you do not need to stay home; rather, you should self-monitor for symptoms and follow the further guidance of the Public Health Office (contacttracing@colorado.edu).

ACCOMMODATION FOR DISABILITIES

If you qualify for accommodations because of a disability, please submit your accommodation letter from Disability Services to your faculty member in a timely manner so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities in the academic environment. Information on requesting accommodations is located on the [Disability Services website](#). Contact Disability Services at 303-492-8671 or dsinfo@colorado.edu for further assistance. If you have a temporary medical condition, see [Temporary Medical Conditions](#) on the Disability Services website.

PREFERRED STUDENT NAMES AND PRONOUNS

CU Boulder recognizes that students' legal information doesn't always align with how they identify. Students may update their preferred names and pronouns via the student portal; those preferred names and pronouns are listed on instructors' class rosters. In the absence of such updates, the name that appears on the class roster is the student's legal name.

HONOR CODE

All students enrolled in a University of Colorado Boulder course are responsible for knowing and adhering to the [Honor Code](#). Violations of the Honor Code may include, but are not limited to: plagiarism, cheating, fabrication, lying, bribery, threat, unauthorized access to academic materials, clicker fraud, submitting the same or similar work in more than one course without permission from all course instructors involved, and aiding academic dishonesty. All incidents of academic misconduct will be reported to Student Conduct & Conflict Resolution (honor@colorado.edu; 303-492-5550). Students found responsible for violating the [Honor Code](#) will be assigned resolution outcomes from the Student Conduct & Conflict Resolution as well as be subject to academic sanctions from the faculty member. Additional information regarding the Honor Code academic integrity policy can be found on the [Honor Code website](#).

SEXUAL MISCONDUCT, DISCRIMINATION, HARASSMENT AND/OR RELATED RETALIATION

CU Boulder is committed to fostering an inclusive and welcoming learning, working, and living environment. University policy prohibits sexual misconduct (harassment, exploitation, and assault), intimate partner violence (dating or domestic violence), stalking, protected-class discrimination and harassment, and related retaliation by or against members of our community on- and off-campus. These behaviors harm individuals and our community. The Office of Institutional Equity and Compliance (OIEC) addresses these concerns, and individuals who believe they have been subjected to misconduct can contact OIEC at 303-492-2127 or email cureport@colorado.edu. Information about university policies, [reporting options](#), and support resources can be found on the [OIEC website](#).

Please know that faculty and graduate instructors have a responsibility to inform OIEC when they are made aware of any issues related to these policies regardless of when or where they occurred to ensure that individuals impacted receive information about their rights, support resources, and resolution options. To learn more about reporting and support options for a variety of concerns, visit [Don't Ignore It](#).

RELIGIOUS HOLIDAYS

Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. In this class, inform the instructor of such conflicts at least three weeks in advance.

See the [campus policy regarding religious observances](#) for full details.

SCHEDULE (TENTATIVE)

The tentative course schedule is as follows (some updates and adjustments may occur during course of the semester):

Week				
No.	Dates	Tuesday	Thursday	Friday
1	1/17-1/19	Lec 1: Introduction to verification and synthesis for stochastic systems motivation – automatic verification and probabilistic model checking, brief case study: FireWire, course overview/organization, reading list, etc.	Lec 2: Discrete-time Markov chains transient and steady state: definitions and computation (incl. BSCCs etc.), fundamental DTMC property, qualitative properties, repeated reachability + persistence	
2	1/24-1/26	Lec 3: Discrete-time Markov chains transient and steady-state: definitions and computation (incl. BSCCs etc.), fundamental DTMC property, qualitative properties, repeated reachability + persistence	Lec 4: Probabilistic temporal logics temporal logic, CTL, PCTL (incl. measurability, qualitative vs. quantitative, equivalences), LTL, probabilistic LTL, PCTL*	HW 1 out
3	1/31-2/02	Lec 5: PCTL model checking for DTMCs [notation, basic algorithm (parse tree), next, bounded until, until, linear equation system solution: direct/iterative, summary/complexity]	Lec 6: The PRISM model checker [basics, language, high-level modelling concepts (e.g. parallel composition), PRISM prop lang specifics (e.g. quant. =? and experiments?), demo (leader election), resources]	Tool Ex. 1 out HW1 due
4	2/07-2/09	Lec 7: Expected costs and rewards [motivation, definitions, PRISM properties, model checking, EGL example (shows rewards since needed for practicals later)]	Lec 8: Continuous-time Markov chains (CTMCs) [continuous probability distributions, exponential distribution + properties, CTMCs, motivating examples (queues, failures, biology), race conditions, embedded DTMC, probability space]	HW 2 out Tool Ex. 1 due
5	2/14-2/16	Lec 9: Continuous-time Markov chains (CTMCs) [steady-state + computation, transient probabilities + uniformization, CSL (syntax, semantics, examples)]	Lec 10: Model checking CTMCs [CSL model checking: (basics, untimed, time-bounded until, S operator), rewards (definition, R operator, model checking summary), CSL model checking summary/complexity]	Tool Ex. 2 out HW 2 due

6	2/21-2/23	Lec 11: Markov decision processes (MDPs) [nondeterminism + uses, MDP defn, adversaries, measures/min/max, adversary types, end components?]	Lec 12: Reachability for MDPs [qualitative case, optimality result, simple adversaries suffice, computation methods: value iteration, linear optimization, policy iteration]	HW 3 out	Tool Ex. 2 due
7	2/28-3/02	Lec 13: PCTL model checking for MDPs [PCTL syntax, semantics, examples, model checking (including precomputation details and re-iteration of reachability stuff), rewards (briefly)]	Lec 14: Automata-based properties [NFAs, DFAs, regular safety properties, DFAs + DTMCs]	Tool Ex. 3 out	HW 3 due
8	3/07-3/09	Lec 15: ω -regular properties [ω -regular, NBAs, DBAs, DRAs, automata + DTMCs]	Lec 16: LTL model checking for DTMCs and MDPs [automata + DTMCs, LTL model checking for DTMCs/MDPs]	HW 4 out	Tool Ex. 3 due
9	3/14-3/16	Lec 17: Strategy synthesis for MDPs [automata + DTMCs, LTL model checking for DTMCs/MDPs]	Lec 18: Case study for MDPs Applications in autonomous driving	Tool Ex. 4 out	HW 4 due
10	3/21-3/23	Lec 19: Probabilistic symbolic model checking BDDs, MTBDDs, operations with BDDs and MTBDDs	Lec 20: Discrete-time, continuous-space stochastic difference equation		Tool Ex. 4 due
11	3/28-3/30	SPRING BREAK			
12	4/04-3/06	Lec 21: Approximate MDP abstraction	Lec 22: Interval-valued MDPs		Final project proposal
13	4/11-4/13	Lec 23: IMDP model checking	Lec 24: Stochastic games	Mid-term out	Mid-term due
14	4/18-4/20	Lec 25: Verification of Stochastic games	Lec 25: Multi-objective MDP		
15	4/25-4/27	Paper presentation	Paper presentation		
16	5/02-4/04	Paper presentation	Paper presentation		Final Project report
	5/09-5/11	FINAL EXAM WEEK			