THE UNIVERSITY OF COLORADO BOULDER

ASEN 6519: Mathematical Foundations of Finite Element Analysis
Fall 2018

SYLLABUS

Instructor: Assistant Professor John Evans
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Office Hours: Open Door Policy / By Appointment

Time: Tuesday/Thursday, 11:00 am – 12:15 pm

Location: KCEN N252

Web Page: Desire2Learn (learn.colorado.edu)

Course Objective:

To establish a fundamental understanding of the mathematical foundations of the finite element method.

Course Overview:

The finite element method is perhaps the most widely used technique in computer aided engineering. While many courses at the University of Colorado Boulder cover how to design and implement a finite element method for applications in structural mechanics, fluid mechanics, and electromagnetics, they typically do not review the mathematical machinery required to study the stability and convergence properties of a finite element method. This course serves to fill that gap. While the course relies heavily on tools emanating from functional analysis, it will not be presumed that students in the course have had a prior course on the topic. Moreover, the course will not cover how to derive major results from Hilbert space, Banach space, and Sobolev space theory. Instead, the course will simply introduce the major results required to study a finite element method and demonstrate how to use these results to conduct a stability, convergence, and error analysis. As such, the course is well-suited for students in aerospace, mechanical, civil, and electrical engineering as well in applied mathematics and computer science conducting research in or with finite elements.

Prerequisites:

There are no formal prerequisites. However, students should be familiar with the finite element method in at least one application context (e.g., fluid mechanics, structural mechanics, electromagnetics), and they should also have a thorough background in partial differential equations. Finally, a familiarity with mathematical analysis would be beneficial.

Required Textbook:

An Introduction to the Mathematical Theory of Finite Elements, Oden and Reddy, Dover, 2011.

Reference and Supplemental Textbooks:

The Finite Element Method, Hughes, Dover, 2000.
The Finite Element Method for Elliptic Problems, Ciarlet, SIAM, 2002.
The Mathematical Theory of Finite Element Methods, Brenner and Scott, Springer, 2008.
Theory and Practice of Finite Elements, Ern and Guermond, Springer, 2004.

Outline of Course:

Part 1: Mathematical Preliminaries

a) Sets, Measure Spaces, Vector Spaces, Normed Linear Spaces, and Inner Product Spaces
b) Definition of a Banach Space and Major Results from Banach Space Theory
c) Definition of a Hilbert Space and Major Results from Hilbert Space Theory
d) Definition of a Sobolev Space and Major Results from Sobolev Space Theory

Part 2: Variational Formulation of Boundary Value Problems

a) Variational Form of Poisson, Stokes, and Advection-Diffusion Equations
b) Strong versus Weak Boundary Conditions
c) The Lax-Milgram Theorem
d) Existence, Uniqueness, and Stability
e) Elliptic Regularity
f) Galerkin Approximations

Part 3: Definition and Properties of a Finite Element Space

a) Construction of Continuous and Discontinuous Finite Element Spaces
b) Interpolation Estimates, Inverse Estimates, and Trace Estimates

Part 4: Finite Element Analysis of Poisson's Equation

a) Galerkin Approximation of Poisson's Equation
b) Consistency, Continuity, and Coercivity
c) Stability, Convergence, and Error Analysis

Part 5: Finite Element Analysis of the Stokes Equation*s*

a) The Inf-Sup Condition
b) Finite Element Pairs Satisfying the Inf-Sup Condition
c) Stability, Convergence, and Error Analysis

Part 6: Finite Element Analysis of the Advection-Diffusion Equation

a) Deficiency of Galerkin's Method
b) Construction of the Streamline Upwind Petrov Galerkin Method
c) Selection of Stabilization Parameters
d) Stability, Convergence, and Error Analysis

Part 7: Weak Enforcement of Boundary and Interface Conditions

a) Weak Boundary Condition Enforcement for Poisson Problems
b) Weak Interface Condition Enforcement for Variable Coefficient Poisson Problems
c) Stability, Convergence, and Error Analysis

Part 8: Potential Advanced Topics

a) *A Posteriori* Error Estimation and Adaptivity
b) Finite Element Analysis of Unsteady Problems
c) Finite Element Analysis of Nonlinear Problems
d) Div- and Curl-Conforming Finite Element Spaces for Darcy and Maxwell Problems
e) Discontinuous Galerkin Methods for Hyperbolic Systems

Class Format:

The class meets twice a week for an hour and fifteen minutes of formal lecture and discussion.

Grading:

40% Homework Assignments
25% Literature Review Project
35% Final Project

Grades will be posted to the class website on Desire2Learn.

Homework Policy:

There will be approximately four to six homework assignments throughout the semester. Students will typically have two to three weeks to complete each assignment. Students should make an effort to turn in assignments that are organized, professional looking, and legible. Students must staple their work (no paperclips or dog-ears). Very messy work will be returned to a student ungraded and a score of zero will be recorded.

Homework is due at the start of class on the due date. Late assignments will not be accepted, though there will be a five-minute grace period. If an on-campus student will be unable to attend class, he or she may submit his or her homework early by slipping it under the instructor's door.

Collaboration is permitted on homework. This means students may discuss the means and methods for solving problems and even compare answers, but students are not free to copy someone's assignment. The work that a student turns in must be his or her own – copying is not allowed for any assignment and will not be tolerated. Students who are caught copying (or providing his or her assignment to another) will receive an "F" for the course and reported to the Dean's office for further punitive action.

Each homework assignment will be given equal weighting in a student's final grade.

Literature Review Project:

A literature review project will be assigned approximately one month into the semester, to be due approximately one month later. For this project, students will review a highly-cited journal article focusing on the mathematical analysis of a finite element method for a particular application (e.g., the Navier-Stokes equations) or a general class of applications (e.g., elliptic partial differential equations). The deliverable of the project will consist of a review paper which summarizes the main contributions of the article, reflects on the impact of the article, and identifies any new insights the article raises.

Final Project:

A final project will be assigned approximately two months into the semester, to be due on the scheduled final examination date. For this project, students will pursue one or three options: (i) design and analyze a finite element method for a particular application of interest, (ii) design and analyze a finite element method for a general class of applications, or (iii) pursue an entirely different direction with permission from the instructor. Students will be allowed to work in teams of two for the final project. The deliverables of the project will consist of both a paper and an in-class presentation. It is highly recommended that students pursue topics that are tightly connected to their own research interest and especially that could possibly result in a refereed journal publication.

Accommodation for Disabilities:

If you qualify for accommodations because of a disability, please submit your accommodation letter from Disability Services to your instructor in a timely manner so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities in the academic environment. Information on requesting accommodations is located on the Disability Services website. Contact Disability Services at 303-492-8671 or dsinfo@colorado.edu for further assistance. If you have a temporary medical condition or injury, see Temporary Medical Conditions under the Students tab on the Disability Services website.

Religious Holidays:

Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. In this class, you must let the instructor know of any such conflicts within the first two weeks of the semester so that he can work with you to make arrangements. See campus policy regarding religious observances for full details.

Classroom and On-Campus Behavior:

Students and faculty each have responsibility for maintaining an appropriate learning environment. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, veteran status, political affiliation or political philosophy. Class

rosters are provided to the instructor with the student's legal name. The instructor will gladly honor your request to address you by an alternate name or gender pronoun. Please advise the instructor of this preference early in the semester so that the instructor may make appropriate changes to his records. For more information, see the policies on classroom behavior and the Student Code of Conduct.

Discrimination and Harassment:

The University of Colorado Boulder (CU Boulder) is committed to fostering a positive and welcoming learning, working, and living environment. CU Boulder will not tolerate acts of sexual misconduct (including sexual assault, exploitation, harassment, dating or domestic violence, and stalking), discrimination, and harassment by members of our community. Individuals who believe they have been subject to misconduct or retaliatory actions for reporting a concern should contact the Office of Institutional Equity and Compliance (OIEC) at 303-492-2127 or cureport@colorado.edu. Information about the OIEC, university policies, anonymous reporting, and the campus resources can be found on the OIEC website.

Please know that faculty and instructors have a responsibility to inform OIEC when made aware of incidents of sexual misconduct, discrimination, harassment and/or related retaliation, to ensure that individuals impacted receive information about options for reporting and support resources.

Honor Code:

All students enrolled in a University of Colorado Boulder course are responsible for knowing and adhering to the Honor Code. Violations of the policy may include: plagiarism, cheating, fabrication, lying, bribery, threat, unauthorized access to academic materials, clicker fraud, submitting the same or similar work in more than one course without permission from all course instructors involved, and aiding academic dishonesty. All incidents of academic misconduct will be reported to the Honor Code (honor@colorado.edu); 303-492-5550). Students who are found responsible for violating the academic integrity policy will be subject to nonacademic sanctions from the Honor Code as well as academic sanctions from the faculty member. Additional information regarding the Honor Code academic integrity policy can be found at the Honor Code Office website.

Prepared by: John Evans Date: August 20, 2018