Actuated Electromagnetic System for Ice Removal

Spring Final Review
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Project Purpose/Objectives
Problem: Ice buildup on aircraft wings in flight
- Decreases Lift-to-Drag Ratio (L/D)
- Reduces mission capabilities
- In extreme cases can result in a crash

Application: ORION Aircraft
- 5 day endurance
- 132 ft. wing span
- Cruising altitude of 20,000-30,000 ft. at 65 kias

Requires: Low mass, low power deicing system to increase flight path possibilities without decreasing capabilities
Problem Statement & Objectives

Design, build, and test a small-scale prototype of a deicing system for the Orion UAV.

Functional Requirements

FR.1 - The full-scale system shall be integrable with the Orion UAV.

FR.2 - The prototype shall remove ice.

FR.3 - The full-scale system shall use less than 4kW-hr to deice the wing section.
Design Description
Design Overview (Principle)

Electromagnetic Deicing Mechanism

1. Capacitor Discharge
2. EM Force
3. Deflection
4. Breaking Ice
Design Overview (Integration)

Solenoid Properties
- 3 inch diameter solenoid
- 60 turns
- 7 mm gap distance
- 3 inch diameter copper target disk

Integrated System = Wing Section + Housing Unit & Support Structure + Deicing Mechanism

- Carbon fiber support structure
- Wing deflection
- Breaking ice
- DAE11 wing section with support structure and solenoid

Mechanism trigger control unit

Purpose/ Objectives  Design Description  Test Overview  Test Results  Systems Engineering  Project Management

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Purpose of Level 3:
- Integration into wing structure-like Orion UAV
- Testing in flight-like wing section and conditions

Ice Testing Occurs in Freezer (\(-15^\circ F\))

- Test cage with viewing window
- Deicing Circuit
- Electric Leaf Blower (3)
- Broken Ice
- Freezing Time
- Test cage with viewing window

Purpose/Objectives
Design Description
Test Overview
Test Results
Systems Engineering
Project Management
Project Roadmap
Project Roadmap

Break Ice

Impulse Required to Break Ice off of Flat Plate

Voltage → Impulse

Solenoid-Impulse

Ballistic Pendulum

Confidence in Solenoid Design

Break Ice off of Flat Plate

Impulse → Deflection

Laser Deflection

Flat Plate Deflection

Confidence in ANSYS Models

Wing Section Deflection

Break Ice off of Wing Section

Legend

Goal

Model

Test

V&V

Conclusions
High Level Model Overview

To remove 3/8 inches of ice off of representative wing section...

**Solenoid-Impulse Model**
- COMSOL - Calculate force based on voltage, solenoid and target disk parameters
- Magnetic Field Lines from Solenoid
- Voltage needed to produce force to break ice

**Flat Plate Deflection Model**
- ANSYS - Calculate expected deflection of carbon fiber flat plate with applied impulse
- Contour plot of flat plate deflection
- Impulse required to break ice off flat plate 0.29 lb-s

**Wing-Section Deflection Model**
- ANSYS - Calculate force required to break ice
- Model that no structural damage occurs with lifetime usage
- Stress plot for wing section
- Impulse required to break ice off wing section 0.26 lb-s
### High Level Test Overview

<table>
<thead>
<tr>
<th>TEST</th>
<th>PURPOSE</th>
</tr>
</thead>
</table>
| Ballistic Pendulum Test | • Verify Solenoid Force Model  
• Refine design using ballistic pendulum test data |
| Laser Deflection Test (Flat Plate) | • Measure deflection to verify material properties via Flat Plate Model |
| Ice Breaking Test (Flat Plate & Wing Section) | • Verify force required to break ice  
• Prove functionality while meeting power and integration requirements |
Levels of Success

Break Ice

Impulse Required to Break Ice off of Flat Plate

Impulse → Deflection

Laser Deflection

Confidence in ANSYS Models

Wing Section Deflection

Confidence in Solenoid Design

Break Ice off of Flat Plate

∧ LEVEL 1 ACHIEVED

Break Ice off of Wing Section

∧ LEVEL 2 ACHIEVED

∧ LEVEL 3 ACHIEVED

Voltage → Impulse

Solenoid-Impulse

Ballistic Pendulum

Legend

Goal
Model
Test
V&V
Conclusions

Purpose/ Objectives

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Test Overview
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Test Results
Level 1: Voltage-Impulse

- **Purpose/Objectives**
  - **Design Description**
  - **Test Overview**
  - **Test Results**
  - **Systems Engineering**
  - **Project Management**

**Legend**
- **Goal**
- **Model**
- **Test**
- **V&V Conclusions**

**Level 1: Voltage-Impulse**

- **Test Overview**
  - **Systems Engineering**
  - **Project Management**

**Goal**
- **Model**
- **Test**
- **V&V Conclusions**

**Legend**
- **Goal**
- **Model**
- **Test**
- **V&V Conclusions**

**Confidence in Solenoid Design**
- **Solenoid-Impulse**
- **Ballistic Pendulum**
- **Break Ice off of Flat Plate**

**Break Ice off of Flat Plate**
- **Wing Section Deflection**

**Break Ice off of Wing Section**
- **Confidence in ANSYS Models**
- **Flat Plate Deflection**

**Impulse Required to Break Ice off of Flat Plate**

**Break Ice**
- **Voltage → Impulse**
- **Impulse → Deflection**

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Level 1- Ballistic Pendulum Test Overview

**Goal:** Verify COMSOL impulse output in order to ensure ballistic pendulum is an adequate tool for measuring impulse

**Method to collect impulse vs. voltage data:**
Measure Max Angle reached by pendulum arm
- Use protractor & high speed camera
- Calculate force/impulse

**Testing Specs = COMSOL Specs**

- Solenoid Outer Diameter Constraint and Gap Distance: \( D = 3 \text{ in}, \ d = 4 \text{ mm} \)
- Solenoid # of Turns Constraint: (COMSOL Software Limitations) \( N = 36 \)

---

**Ballistic pendulum test setup**

---

<table>
<thead>
<tr>
<th>Reqs Verified with Test</th>
<th>DR.3.1 Operate on an incoming 28 V DC voltage line.</th>
<th>DR.3.2 Instantaneous power draw shall be at most 2 kW.</th>
<th>DR.2.1 Be capable of removing 3/8 inch thick ice on test section</th>
</tr>
</thead>
</table>

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**Purpose/ Objectives**

**Design Description**

**Test Overview**

**Test Results**

**Systems Engineering**

**Project Management**
Level 1- Ballistic Pendulum Test Results

COMSOL Model Verification

Testing Specs = COMSOL Specs
- Solenoid outer diameter = 3 in
- Solenoid inner diameter = 0.25 in
- Gap distance = 4 mm
- Number of turns = 36

Conclusions
- Model-predicted impulse matches test results
- Modeling software limitations- based on experimental data trends, solenoid design was improved upon

Implications of Model Verification
- Verification gives confidence in test data
- Test data becomes modeling tool (model is geometrically limited)

890V required to produce impulse to break ice off flat plate (0.29 lb-s) with model-limited solenoid
Level 1- Solenoid Design Refinement

Improved Solenoid Design
- Solenoid outer diameter = 3 in
- Solenoid inner diameter = 0.25 in
- Gap distance = 4 mm
- Number of turns = 60

Conclusions for Refined Model
- 60-turn solenoid produces greater impulse at less voltage
- Energy-consumption is greater concern over mass consumption
  - 36 Turns → 198 J
  - 60 Turns → 126 J
  - → 36% Energy Savings by using 60 turns vs. 36 turns

710V required to produce impulse to break ice off flat plate (0.29 lb-s) with max number of turns solenoid

Level 1 Success Conclusions:
- ✓ Mechanism produces impulse required to break ice
- ✓ Energy consumption = 126 J

✓ DR.2.1
✓ DR.3.2
Level 1: Achieved

Break Ice

Impulse Required to Break Ice off of Flat Plate

Voltage \rightarrow Impulse

Solenoid-Impulse

Ballistic Pendulum

Confidence in Solenoid Design

\checkmark LEVEL 1 ACHIEVED

Break Ice off of Flat Plate

Impulse \rightarrow Deflection

Laser Deflection

Flat Plate Deflection

Confidence in ANSYS Models

Break Ice off of Wing Section

Wing Section Deflection

Legend

Goal
Model
Test
V&V
Conclusions

Purpose/Objectives
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Level 2: Impulse-Deflection

Voltage $\rightarrow$ Impulse

Solenoid-Impulse $\rightarrow$ Ballistic Pendulum

Confidence in Solenoid Design

Break Ice off of Flat Plate

Impulse Required to Break Ice off of Flat Plate

Impulse $\rightarrow$ Deflection

Laser Deflection $\rightarrow$ Flat Plate Deflection

Confidence in ANSYS Models

Wing Section Deflection

Break Ice off of Wing Section
Level 2- Flat Plate Deflection Test Overview

Goal: Verify ANSYS force model through deflection measurements

Method to measure surface deflection
- Altered geometry from actuation
  - Reflected laser displacement
- High speed camera
  - Long exposure against ruler

Predicted flat plate deflection* = 0.3 in

*Corresponds to measureable deflection without ice at force required to break ice

DR.1.3 Operation shall **not damage or degrade** wing

DR.2.1 Be capable of **removing 3/8 inch thick ice** on test section
Test conditions match Flat Plate Model conditions

- Boundary conditions = 8 fixed points (corners & mid-sides)
- Impact location same in ANSYS and test

Recall
0.29 lb-s = Impulse required to break 3/8 inches of ice off flat plate
Modeled as pressure applied over target disk area

ANSYS Flat plate deflection model with Impulse = 0.29 lb-s
Level 2- Flat Plate Model Refinement

Level 2 Deflection Test Conclusions:

- Refined material properties for further confidence in models (ice breaking predictions)

- Carbon fiber deflects enough from mechanism impulse to theoretically break ice

Predicted (extrapolated) deflection measurement (no ice) at impulse required to break ice off flat plate = 0.092 in + 0.014 in

Refinement

- Carbon Fiber Young's Modulus
  - Starting value = 61340 MPa
  - Refined value = 213400 MPa
- Original value based on research, new value from actual material
Level 2- Flat Plate Ice Removal Test Results

Purpose: check functionality of ice breaking on simple geometry

Testing conditions
• 3/8 in ice thickness
• -15°F ambient temperature
• Actuated at 615V

• First Blast: Removed ~50% of the ice.
  - After blast #1: Cracks had fully propagated through the ice.
• Second Blast: Removed an additional ~45%.

Level 2 Deflection Test Conclusions:
✓ Refined material properties for further confidence in models (ice breaking predictions)
✓ Carbon fiber deflects enough from mechanism impulse to theoretically break ice
Level 2: Achieved

Break Ice

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Confidence in Solenoid Design

Ballistic Pendulum

Impulse → Deflection

Laser Deflection

Confidence in ANSYS Models

Flat Plate Deflection

Break Ice off of Flat Plate

Wing Section Deflection

Break Ice off of Wing Section

✓ LEVEL 2 ACHIEVED ← →
Level 3: Integration & Functionality

- **Break Ice**
  - Impulse Required to Break Ice off of Flat Plate
  - Voltage → Impulse
    - Solenoid-Impulse → Confidence in Solenoid Design
    - Ballistic Pendulum → Break Ice off of Flat Plate
  - Impulse → Deflection
    - Laser Deflection → Flat Plate Deflection
    - Confidence in ANSYS Models
    - Wing Section Deflection → Break Ice off of Wing Section

Legend:
- Goal
- Model
- Test
- V&V
- Conclusions

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Level 3- Wing Section
Test Overview

**Goal:** Proof of functionality while meeting design requirements.

### Testing Environment
- **Location:** walk-in freezer at INSTAAR
- **Testing temperature range:** -15ºF → 0ºF
- **Wind speed:** 65 knots average (at leading edge)

### Testing Procedure
- **Setup wing section to cast ice (~ 4 hrs)**
- **Prepare wing section in wind cage (& leaf blowers) for testing**
- **Transport mechanism, power supply into freezer**
- **Turn on leaf blowers, actuate mechanism with flat plate/full wing section**
- **If ice remaining, charge & actuate until clear**

---

**Reqs Verified with Test**
- **DR.1.2** Deicing mechanism shall be integrable with DAE11-shaped wing
- **DR.2.1** The deicing mechanism shall remove 3/8-inch thick ice
- **DR.2.3** Max thickness of ice remaining = 0.1 inches

---

Wind cage, wind speed, test section setup in walk-in freezer
Model Properties

**Boundary Conditions**
- Fixed at the spar

**Modulus Values**
- \( E \) for carbon fiber = 61.34 GPa
- \( E \) for nomex honeycomb = 255 MPa

Integrated Mechanism Properties

**Modulus Values**
- Solenoid Diameter = 3 inches
- Target Disk Diameter = 3 inches
- Gap Distance = 7 mm

**Required Impulse from ANSYS to break ice off WING SECTION** = 0.26 lb-s

**Actuate mechanism at (minimum) 770V to break ice.**
Level 3- Wing Section
Ice Removal Test Results

**Purpose/Objectives**

- **Design Description**
- **Test Overview**
- **Test Results**
- **Systems Engineering**
- **Project Management**

**Test Overview**

**Initial**

**Impulse #1**

**Impulse #2**

**Impulse #3**

**Testing done at 612 V**

**ANSYS predicted a Impulse of 0.29 lb-s**

This is equivalent to 710 V

Testing done at 612 V

Initial

Impulse #1

Impulse #2

Impulse #3

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900 Volts = 0.35 lb-s

- First Blast: Removed ~80% of the ice.
- Second Blast: Removed all remaining big chunks.

900 Volts only required 2 blasts

After blast #1: Cracks had fully propagated through the ice.
Level 3 - Wing Section
Ice Removal Test Results

Summary of Results:

- Mechanism successfully broke ice → Proof of functionality
- Higher voltages → Fewer impulses needed
- Ice removal hindered by adhesion → Should be modeled in the future
- Remaining ice had a depth of > 0.1 in → May disrupt laminar flow

Level 3 Ice Removal Requirement Summary:

- DR.1.2 ✓ System successfully integrated within DAE11 test section
- DR.2.3 X Maximum ice thickness after actuation was greater than 0.1 in.
- DR.2.1 ✓ The deicing mechanism shall be capable of removing 3/8 in thick ice on test section.
Level 3: Integration & Functionality

Break Ice

Impulse Required to Break Ice off of Flat Plate

Voltage → Impulse

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Ballistic Pendulum

Confidence in Solenoid Design

Break Ice off of Flat Plate

Impulse → Deflection

Laser Deflection

Flat Plate Deflection

Confidence in ANSYS Models

Wing Section Deflection

Break Ice off of Wing Section

✓ LEVEL 3 ACHIEVED
Conclusions from Levels

- Recap solenoid selection
- Flat Plate Model refinement based on material properties
- Requires refinement of Wing Section Model based on refined material properties and on ice adhesion

Lessons Learned:

- LEVEL 1 ACHIEVED
- Break Ice off of Flat Plate
- LEVEL 2 ACHIEVED
- Break Ice off of Wing Section
- LEVEL 3 ACHIEVED
- ALL 3 LEVEL ACHIEVED
From testing, 1 Solenoid clears 2 ft. section of ice off wing section

→ For full-span, deicing requires 62 solenoids + Housing + Supporting Circuitry

Total Mass Estimate = 200 lb.

Total Power Estimate = 310 W to recharge and fire at 5 minute intervals

Note: requires further testing to account for extra rigidity of ORION wing ribs and further testing on ice crack and shed areas
Systems Engineering

Fall Semester:
- Project Understanding
- Modeling & Feasibility

Spring Semester:
- Manufacturing
- Model Verification
### Fall Semester

#### Major Tasks
- Gain scope of project
- Determine Levels of Success
- Develop requirements to accomplish scope

#### Major Difficulties
- Customer was vague about project desires
- Hard to put numbers to parts of project

#### Major Tasks
- Model required force to break ice
- Model solenoid force

#### Major Difficulties
- Figuring out model for solenoid
- Distributing tasks among team
- Solving design choice and not changing
**Spring Semester**

<table>
<thead>
<tr>
<th>Major Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Manufacture tests for levels of success</td>
</tr>
<tr>
<td>- Perform tests</td>
</tr>
<tr>
<td>- Build models for interpreting test data</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Major Difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Building ballistic pendulum</td>
</tr>
<tr>
<td>- Scheduling for shipping</td>
</tr>
<tr>
<td>- Capturing Laser Deflection</td>
</tr>
</tbody>
</table>

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**Systems Engineering**

<table>
<thead>
<tr>
<th>Operations and Maintenance</th>
<th>System Validation</th>
<th>System Verification &amp; Deployment</th>
<th>Subsystem Verification</th>
<th>Unit/Device Testing</th>
<th>Software / Hardware Development</th>
<th>Field Installation</th>
</tr>
</thead>
</table>

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**Purpose/Objectives**

- Design Description
- Test Overview
- Test Results
- Project Management

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Lessons Learned

**Fall Semester:**
- Don’t lean on customer for whole project scope.
- **REALLY** know project before moving forward.
- Engineers model then validate.

**Spring Semester:**
- Don’t expect to get it right the first time it’s re-built.
- Shipping takes 2X longer than expected.
- Shipping costs 2X more than expected.
Project Management
Project Management

Successes

~ Early planning for testing accommodations
~ Execution for all 3 levels of success
~ Team dynamic & communication

Challenges

~ Defining project
~ Keeping progress high when project is at a low
~ Maintaining communication
~ Consistent distribution of tasks

LESSONS LEARNED

• Margin is critical – in both TIME and BUDGET
• Communication & passion are the driving forces behind team success
• It is physically possible to break ice using electromagnetism
# Budget Comparison

**Budget:**
- **Aerospace Department:** $5,000
- **EEF:** $2,215
- **Total Available Budget:** $7,215

## CDR Budget
- **Margin:** $2,017
- **Electronics:** $1,892
- **Wing Test Section:** $1,001
- **Management:** $90
- **Ice Casting Trough:** $175

## Actual Budget
- **Electronics:** $2,256
- **Dynamic Testing:** $1,653
- **Wing Test Section:** $2,293
- **Management:** $394
- **Margin:** $444

**Total Expenses:** $6,771 (94%)

**Remaining Budget:** $444

**Unforeseen Expenses:**
- Useless $700 fan
- Sophisticated mechanism assembly
- Layup Materials
- Ballistic Pendulum
- Leaf Blowers
- Printing & poster costs

**Future Purchases:**
- Printing Project Final Report
### Industry Cost

**Total Team Hours = 3,685**

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team Hours</td>
<td>$115,156</td>
</tr>
<tr>
<td>Including 200% overhead cost</td>
<td>$115,156</td>
</tr>
<tr>
<td>Material Cost</td>
<td>$6,771</td>
</tr>
</tbody>
</table>

**Total Industry Cost:** $237,083

**Assumes $65k salary for each team member**
An electromagnetic deicing system is a **Viable** solution for deicing the Orion UAV.
Questions?
References


Critical Project Elements

**Ballistic Pendulum**
- Pendulum Assembly

**Wing Section**
- Test Section
- Housing Unit & Support Structure

**Test Setup**
- Ice Casting
- Wind Speed & Test Cage

- Ballistic Pendulum
- Wing Section
- Test Setup

- Purpose/Objectives
- Design Description
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Functional Block Diagram

Electrical System

- Charge
- Discharge
- Dump

Charging Circuit

Discharging Circuit

1000 V Power Supply

3.5 V Power Supply

Solenoid

Target Disk

Deicing Mechanism

Operator (person)

Legend

- Mechanism
- Electrical
- Operation
- Low Voltage Line
- High Voltage Line
- Force Interaction
- Operator Control

Purpose/ Objectives

Design Description

Test Overview

Test Results

Systems Engineering

Project Management

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## Requirements – FR1

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FR.1</strong></td>
<td>The full-span system shall be integrable with the Orion UAV.</td>
</tr>
<tr>
<td><strong>DR.1.2</strong></td>
<td>The deicing mechanism shall be integrable with a wing in the shape of the DAE11 airfoil.</td>
</tr>
<tr>
<td><strong>SPEC.1.2.1</strong></td>
<td>The test section chord length shall be 72 in (6 ft).</td>
</tr>
<tr>
<td><strong>DR.1.2.1</strong></td>
<td>The components of the deicing mechanism internal to the wing test section shall fit between the leading edge (0 in.) and half-chord line (36 in.) in the chord-wise direction.</td>
</tr>
<tr>
<td><strong>DR.1.3</strong></td>
<td>The installation of the deicing mechanism shall not damage or degrade the structural integrity of the wing.</td>
</tr>
<tr>
<td><strong>DR.1.4</strong></td>
<td>The operation of the deicing mechanism shall not damage or degrade the structural integrity of the wing over a lifetime of 150 hours.</td>
</tr>
</tbody>
</table>
FR.2 The deicing mechanism shall remove ice.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPEC.2.1</td>
<td>The deicing mechanism shall remove ice in an environment with wind speed = 65 knots.</td>
</tr>
<tr>
<td>DR.2.1</td>
<td>The deicing mechanism shall be capable of removing 3/8 in thick ice on test section.</td>
</tr>
<tr>
<td>SPEC.2.1.1</td>
<td>The ice shall cover the test section from the leading edge to 7% of the chord (7.2 in) as measured chord-wise from the leading edge on the upper airfoil surface and to 2% of the chord (1.7 in) as measured chord-wise from the leading edge on the lower airfoil surface.</td>
</tr>
<tr>
<td>DR.2.2</td>
<td>The deicing mechanism shall be capable of removing ice at any time during a five-day continuous flight.</td>
</tr>
<tr>
<td>DR.2.3</td>
<td>The maximum allowable thickness of ice remaining at any point along the surface of the test section after activating the prototype shall be 0.1 in.</td>
</tr>
</tbody>
</table>
FR.3 The full-span system shall use less than 4kW-hr of energy to deice the wing section.

| DR.3.1 The deicing mechanism shall operate on an incoming 28 V DC voltage line. |
| DR.3.2 The full-span system instantaneous power draw shall be at most 2 kW. |
Backup - TRR Schedule

Legend:
- Ballistic Pendulum
- Flat Plate
- Full Wing Section
- AIAA
- Symposium
- SFR
- Testing
- Margin
- Milestone
- Completion

Purpose/ Objectives
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Backup - CDR Cost Plan

Manufacturing
- Test Section
- Housing Unit
- Ice Cast Mold

Electronics
- Circuit
- Solenoid

Management
- Gantt Chart

Total Expenses: $2,983
Margin: $2,017
Deflection Measurement

\[ d = r \sqrt{\sin(\varphi) \sum_{k=1}^{P} \sin[(2k - 1)\varphi]} \]

\[ = \frac{1}{2\tan(\theta)} \left[ D - \frac{2L}{2\cos^2(\theta) \cot(2\varphi) + \sin(2\theta)} \right] \]
\[ d = 2R \sin(\varphi) \sum_{k=1}^{P} \sin[(2k - 1)\varphi] \]
\[ r = 2R \sin(\varphi) \sum_{k=1}^{P} \cos[(2k - 1)\varphi] \]
\[ R = \frac{r^2}{2d} \]
Backup Slides for Equation

\[
D = D_1 + D_2 = 2d \tan(\theta) + \frac{L \sin(2\varphi)}{\cos(\theta) \sin\left(\frac{\pi}{2} + \theta - 2\varphi\right)}
\]

\[
d = \frac{1}{2\tan(\theta)} \left[ D - \frac{L \sin(2\varphi)}{\cos(\theta) \sin\left(\frac{\pi}{2} + \theta - 2\varphi\right)} \right]
\]

\[
= \frac{1}{2\tan(\theta)} \left[ D - \frac{2L}{2\cos^2(\theta) \cot(2\varphi) + \sin(2\theta)} \right]
\]
Switch from Avg. Force to Impulse

- We cannot apply the exact waveform applied by our solenoid in ANSYS. And because the time is short, impulse will better account for the differences.

- Average force is deceptive. It is completely possible to have a higher overall average force, but be less effective.

- Reduces error due to time assumptions. Our current average force models make assumptions for discharge time. Using impulse removes these assumptions.
Level 1- Ballistic Pendulum

Impulse Calculations

\[ PE = mgh = mg[L_{com}(1 - \cos\theta)] \]

\[ \omega = \sqrt{\frac{2PE}{I}} \]

\[ V_{com} = \omega \times L_{com} \]

\[ \text{Impulse} = V_{com} \times m \]
Fatigue

\[ \sigma_{\text{max}} = 207 \text{ MPa} \]

Stress in wing under normal flying conditions:

\[ \epsilon = 1500 \mu \]

\[ \sigma_{\text{min}} = E\epsilon = (41 \text{ GPa})(1500 \mu) = 61 \text{ MPa} \]

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = 45.5 \text{ MPa} \]

Goodman's Relation:

\[ \sigma_a = \sigma_f \left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) = 425\text{MPa} \left(1 - \frac{45.5\text{MPa}}{500\text{MPa}}\right) \]

\[ \sigma_{a,\text{max}} = 386 \text{ MPa} \quad \text{Maximum allowable stress amplitude} \]

\[ \sigma_{a,\text{actual}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = 73 \text{ MPa} \]

✓ Actual stress amplitude is less than maximum