MCEN 5023/ASEN 5012

Chapter 7

**Linear Elasticity – Plane Problems**

Fall, 2006
Plane Problems

Plane Stress and Plane Strain:

Plane stress: All the stress components associate with 3-direction are zero.

\[ \sigma_{33} = \sigma_{13} = \sigma_{23} = 0 \quad \sigma_{11} \neq 0 \quad \sigma_{22} \neq 0 \quad \sigma_{12} \neq 0 \]

\[ F_3 = 0 \]

Plane strain: All the strain components associate with 3-direction are zero.

\[ e_{33} = e_{13} = e_{23} = 0 \quad e_{11} \neq 0 \quad e_{22} \neq 0 \quad e_{12} \neq 0 \]

\[ u_3 = 0 \]
Plane Problems

Plane Stress and Plane Strain:

In a plane stress problem, which components of strain are non-zero?

In a plane strain problem, which components of stress are non-zero?
Plane Problems

Plane Stress and Plane Strain:

X3 direction dimension is infinitesimal

Plane Stress
Plane Problems

Plane Stress and Plane Strain:

The two ends (X3 surfaces) of the bar are fixed

Plane Strain
Plane Problems

Plane Stress and Plane Strain:

Dam

Tunnel
Plane Problems

Plane Stress and Plane Strain:

In general, if the problem has one dimension is much larger (or smaller) than the other two directions, one should consider plane strain (stress).
Plane Problems

Under what conditions a problem can be approximated as a plane problem?

Geometry:
the cross section of the body is independent of $X_3$ coordinate

If two ends are fixed?

Yes

No

If $X_3$ dimension is very thin?

Yes

No

If the acting zoom of load is small as compared with the characteristic length in $X_1$-$X_2$ plane?

No

Yes

Plane Stress

Plane Strain
Plane Problems

Plane Stress and Plane Strain:
Plane Problems

Plane Stress and Plane Strain:

**Plane Stress**

\[ E^* = \frac{E}{1 - \nu^2} \quad \nu^* = \frac{\nu}{1 - \nu} \]

**Plane Strain**

\[ E' = E \left(\frac{1 + 2\nu}{(1 + \nu)^2}\right) \quad \nu' = \frac{\nu}{1 + \nu} \]
Plane Problems

Plane Problems:
Plane Problems

Solutions to Plane Problems

1. Displacement Methods

2. Stress Methods

3. Stress Function (Airy Stress Function)
Plane Problems

Airy Stress Function
Plane Problems

Airy Stress Function
Airy Stress Function: Short Beam

\[
\phi = A(a - x)(y^3 - 3b^2y)
\]

P is total shear force on the left side
Plane Problems

Airy Stress Function: Short Beam
Plane Problems

Airy Stress Function: Short Beam

\[ \sigma_{11} = 6A(a - x)y \]

\[ \sigma_{12} = A(3y^2 - 3b^2) \]

\[ A = -\frac{P}{4hb^3} \]
Plane Problems

Airy Stress Function: Short Beam

- Parabolic distribution
- Zero at top/bottom surface
Plane Problems

Polar Coordinates

\[ r, \theta, e_r, e_\theta \]
Plane Problems

Airy Stress Function: Axisymmetric Problem

\[ \phi = \phi(r) \]
Plane Problems

Pressure Vessel:

Inner Diameter: 2a
Outer Diameter: 2b
Plane Problems

Pressure Vessel:
Pressure Vessel

\[ p_i = 1, \quad p_o = 0 \]
Plane Problems

Application of Pressure Vessel:

Infinite Plate with a small central hold subjecting to equibiaxial tension
Plane Problems

Airy Stress Function: Non-axisymmetric Problem

Stress concentration:
Plane Problems

Airy Stress Function: Non-axisymmetric Problem

Stress concentration:
Plane Problems

Airy Stress Function: Non-axisymmetric Problem

Stress concentration:
Plane Problems

Airy Stress Function: Non-axisymmetric Problem

Stress concentration:
**Plane Problems**

Airy Stress Function: Non-axisymmetric Problem

\[ p_1 = 1, \quad p_2 = 0 \]

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