1. Ashcroft & Mermin 4.1. (This notation, which we will continue to use, means problem number 1 in chapter 4.)

2. **FCC Lattice as stacked triangular layers.** The HCP lattice is a lattice with basis, where the Bravais lattice primitive vectors are

\[
a_1 = ax \tag{1}
\]
\[
a_2 = \frac{a}{2} x + \frac{a\sqrt{3}}{2} y \tag{2}
\]
\[
a_3 = cz, \tag{3}
\]

and the basis vectors are

\[
c_1 = 0 \tag{4}
\]
\[
c_2 = \frac{1}{3} a_1 + \frac{1}{3} a_2 + \frac{1}{2} a_3. \tag{5}
\]

In Ashcroft & Mermin Chapter 4 (p. 76-79), and also in lecture, it is explained how this lattice can be viewed as an “ABABAB” stacking of triangular layers, stacked in the z-direction.

Suppose we consider instead the Bravais lattice obtained by replacing \(a_3\) with \(a'_3 = c_2\). Show that this gives an “ABCABCABC” stacking of triangular layers. Show that this lattice is the FCC lattice, for a particular choice of \(c\). What is this special value of \(c\)?

3. **Rotational symmetry of 2d Bravais lattices.** There are very strong constraints on the symmetry properties of Bravais lattices – a nice illustration of this is provided by rotational symmetry in 2d Bravais lattices. (In this problem, you should use only basic properties of Bravais lattices. You may not appeal to the fact that there are only 5 distinct kinds of 2d Bravais lattices.)

(a) Consider a 2d Bravais lattice, and suppose it has an \(n\)-fold rotational symmetry. (This means the lattice is symmetric under a rotation by an angle \(2\pi/n\) about some point \(r_0\).) Suppose that \(r_0\) is not a point in the Bravais lattice. Prove that there must also be an \(n\)-fold rotational symmetry about some other point \(r'_0\), which is a Bravais lattice point. Finally, prove that there must be an \(n\)-fold symmetry about every Bravais lattice point.

(b) Prove that it is impossible to have an \(n\)-fold rotational symmetry where \(n = 5\) or \(n \geq 7\).

(c) Show that the only 2d Bravais lattice with a 4-fold rotational symmetry is the square lattice.

(d) Show that the only 2d Bravais lattice with a 3-fold rotational symmetry is the hexagonal lattice. Since 6-fold symmetry implies 3-fold symmetry, this means that the hexagonal lattice is also the only 2d Bravais lattice with 6-fold rotational symmetry.

4. (a) Consider a plane that contains any three non-collinear points of a 3d Bravais lattice. Show that this plane must contain an entire 2d Bravais lattice, made up of points from the original 3d lattice. Such a plane is referred to as a lattice plane of the 3d lattice.

(b) Suppose a 3d Bravais lattice has an \(n\)-fold rotational symmetry. Use the results of 3(b) and 4(a) to prove that \(n = 5\) and \(n \geq 7\) are also impossible for 3d lattices.