1. Calculate the occupancy of the first excited state of a Bose-condensed system in a three-dimensional harmonic trap. Show that this state is not macroscopically occupied for any temperature, i.e. that the fraction of bosons in this state is always zero in the thermodynamic limit.

2. Pathria and Beale 7.19.

   (a) Calculate the Bose–Einstein condensation temperature for a system with energy density of states given by
   \[ g(\varepsilon) = \frac{\alpha}{\epsilon_1} \left( \frac{\varepsilon}{\epsilon_1} \right)^{\sigma}, \]
   for one-particle energy scale \( \epsilon_1 \), power \( \sigma \), and constant \( \alpha \). Express the critical temperature in terms of \( \epsilon_1 \), \( \sigma \), \( \alpha \), \( N \) and the \( \zeta \)-function. Show that the critical temperature tends to zero as \( \sigma \to 0 \).
   (b) Now calculate the energy density of states for a \( d \)-dimensional harmonic oscillator. Relate the parameters \( \epsilon_1 \), \( \sigma \), and \( \alpha \) to \( \hbar \omega_0 \) and \( d \). Treating \( d \) as a continuous parameter, find the \( d \) and \( N \)-dependence of the condensation temperature as \( d \to 1 \).
   (c) Determine the analytical behavior of the condensate fraction and heat capacity below the condensation temperature and behavior of the constant-\( N \) heat capacity below the condensation temperature for \( d \) close to 1.

4. Pathria and Beale 7.21

5. Pathria and Beale 7.23

6. Pathria and Beale 7.24