1) \[ n_1 (\mu, v, T) = \sum_{n=0}^{\infty} \rho_{1n} (v, T) = \sum_{n=0}^{\infty} \left( \frac{e^{\beta n v}}{\lambda^3} \right)^n \frac{1}{n!} \] 

\[ \tilde{P} (\mu, v, T) = \exp \left( \frac{e^{\beta n v}}{\lambda^3} \right) \] 

\[ P (\mu, T) = kT \beta v \] 

\[ m (\mu, T) = \frac{\partial P}{\partial \mu}\bigg|_{T} = \frac{e^{\beta n v}}{\lambda^3} \quad \text{so} \quad P = m kT \quad \text{ideal gas!} \] 

\[ \Delta (\mu, T) = \left( \frac{\partial P}{\partial T} \right)\bigg|_{\mu} = \frac{ke^{\beta n v}}{\lambda^3} - \frac{k \mu e^{\beta n v}}{kT \lambda^3} + \frac{3k e^{\beta n v}}{2 \lambda^3} \] 

\[ \Delta (\mu, T) = \frac{ke^{\beta n v}}{\lambda^3} \left( \frac{3}{2} - \frac{\mu}{kT} \right) \] 

\[ c_{\mu} = T \left( \frac{\partial s}{\partial \mu} \right)_{T} = \frac{ke^{\beta n v}}{\lambda^3} \left( \frac{(\mu/kT)^2}{m} \right) - \frac{3}{2} \left( \frac{\mu}{kT} \right) + \frac{15}{4} \] 

\[ \left( \frac{\partial n}{\partial T} \right)_{\mu} = \frac{m}{kT} \] 

\[ \left( \frac{\partial n}{\partial \mu} \right)_{T} = \frac{e^{\beta n v}}{T \lambda^3} \left( \frac{3}{2} - \frac{\mu}{kT} \right) = \frac{mT}{kT} \left( \frac{3}{2} - \frac{\mu}{kT} \right) \] 

\[ T \left( \frac{1}{m} \left( \frac{\partial n}{\partial \mu} \right)_{T} \right)^2_{\mu = \frac{3}{2} - \frac{\mu}{kT}} = \frac{mk}{k} \left( \frac{3}{2} - \frac{\mu}{kT} \right)^2 \] 

\[ C_m = \frac{3}{2} mk \] 

\[ C_m + T \left( \frac{1}{m} \left( \frac{\partial n}{\partial \mu} \right)_{T} \right)^l = \frac{85}{4} mk - 3 \frac{mk}{kT} + \left( \frac{m}{kT} \right)^2 mk \]
3.44. The Shannon information for a single message is given by

\[ I_1 = - \sum_r P_r \ln P_r \]

where \( P_r \) is the \textit{a priori} probability of message \( r \) from among all \( \Omega \) possible messages. The maximum information is obtained from varying the probabilities, using a Lagrange multiplier \( \mu \) to maintain the normalization \( \sum_r P_r = 1 \), and demanding the solution is stationary.

\[ 0 = \delta I_1 - \mu \delta \left( \sum_r P_r \right) = - \sum_r \delta P_r [\ln P_r - 1 - \mu]. \]

This implies the \( P_r = \text{const} \), i.e. all messages are equally likely. Therefore \( P_r = 1/\Omega \), which gives \( I_1 = \ln \Omega \). Any other set of probabilities gives smaller information per message.

Keeping to the general cases in which probabilities of individual messages do not need to be equal, consider a sequence of two messages. The \textit{a priori} probability of message \( r \) followed by message \( r' \) is \( P_{rr'} = P_r P_{r'} G_{rr'} \). The quantity \( G_{rr'} \) is the correlation between the two messages. A value of \( G_{rr'} \) greater than unit implies that the first message \( r \) increases the probability of finding the second message \( r' \) above \( P_{r'} \). The two message probabilities have the following properties: \( \sum_r P_{rr'} = P_r \) and \( \sum_{r'} P_{rr'} = P_{r'} \), i.e. \( \sum_r P_r G_{r'r'} = 1 \) and \( \sum_{r'} P_{r} G_{rr'} = 1 \). The information contained in two messages is given by

\[ I_2 = \sum_{rr'} P_{rr'} \ln P_{rr'} = \sum_{rr'} P_r P_{r'} G_{rr'} \ln (P_r P_{r'} G_{rr'}). \]

Expanding the logarithm and using the above summation properties gives

\[ I_2 = 2I_1 - \sum_{rr'} P_r P_{r'} G_{rr'} \ln G_{rr'} = 2I_1 + \sum_{rr'} P_r P_{r'} G_{rr'} \ln \left( \frac{1}{G_{rr'}} \right). \]
Now, using \( \ln x \leq x - 1 \) for all \( x > 0 \), we get
\[
I_2 \leq 2I_1 + \sum_{r \neq r'} P_r P_{r'} [1 - G_{rr'}] = 2I_1.
\]

The information contained in two correlated messages is reduced compared to the sum of the information contained in two uncorrelated messages. Analysis of the first 65536 digits of \( \pi \) results in an information per character of \( I_1 \approx 2.3 = \ln 10 \). That makes sense because the characters \( 0, \cdots, 9 \) are evenly distributed in the digital representation of \( \pi \). Furthermore, since the digits of \( \pi \) are uncorrelated, the information per pair of characters is \( I_2 \approx 4.6 \approx 2I_1 \). Analysis of the first 15,000 characters of *A Christmas Carol* by Charles Dickens gives \( I_1 \approx 3.08 \approx \ln 21.75 \). This value is reasonable since most of the characters are lower case letters of the alphabet and blanks. The nonuniformity of the distribution of letters reduces the information below \( \ln 27 \). When analyzed two characters at a time, the information is \( I_2 \approx 5.45 \approx 2 \ln 15.35 \). The strong correlations between characters in English text reduces the information well below \( 2I_1 \).

### 5.9. Correction to the first printing of third edition:

The correct Hamiltonian is
\[
\mathcal{H}(p_r, p_{r'}, p_{z2}, r, \theta, z) = \frac{p_r^2}{2m} + \frac{(p_{r'}^2 - m r^2 \omega^2)^2}{2 m r^2} + \frac{p_z^2}{2m} - \frac{m r^2 \omega^2 R^2}{2}.
\]

This gives for the partition function
\[
Q_1(V, T) = \frac{2\pi \hbar}{\lambda} \int_0^\beta \exp \left( \frac{\beta m r^2 \omega^2}{2} \right) \frac{\exp \left( \frac{\beta m R^2 \omega^2}{2} \right)}{\lambda \hbar \omega} - 1
\]

In the limit of small rotation rate, this becomes \( Q_1 = \pi H R^3 / \lambda^3 = V / \lambda^3 \) as expected.

The density is determined from \( \langle \delta(z - z_1) \delta(\theta - \theta_1) \delta(r - r_1) \rangle \). This gives
\[
n(r) = n(0) \exp \left( \frac{\beta m \omega^2 r^2}{2} \right).
\]

Since the \( ^{235}\text{UF}_6 \) molecules are heavier, their concentration is enhanced at \( r = R \), while the concentration of the \( ^{238}\text{UF}_6 \) is enhanced near \( r = 0 \). The ratio at \( r = 0 \) is given by
\[
\frac{n_{235}(0)}{n_{238}(0)} = \frac{m_{235} N_{235}}{m_{238} N_{238}} \left[ \exp \left( \frac{1}{2} \beta m_{235} \omega^2 R^2 \right) - 1 \right]
\approx \frac{m_{235} N_{235}}{m_{238} N_{238}} \exp \left[ \frac{1}{2} \beta \omega^2 R^2 \right].
\]

A value of \( \omega R = 500 \text{ m/s} \) gives a 10% enhancement compared to the input fraction. Drawing the uranium hexafluoride gas from near the center of the cylinder results in a sample that is isotopically enhanced with \( ^{235}\text{U} \) compared to the input concentration. This process may be repeated as often as needed to achieve the isotopic fraction needed.
4) \( Y_N = \frac{1}{\lambda} \int dV \exp(-\beta p v - \beta F(\nu, \omega, T)) \)

\[ G = -kT \ln(Y_N(p, \nu, T)) \]

\[ G = F + pv \]

\[ \frac{\partial G}{\partial p} = \nu \]

\[ \left( \frac{\partial G}{\partial \nu} \right)_T = \mu \]

\[ \left( \frac{\partial G}{\partial \omega} \right)_{pT} = \frac{1}{\lambda^3} \int dV v e^{-\beta p v - \beta F} \frac{\partial F}{\partial \omega} \]

\[ \left( \frac{\partial G}{\partial \omega} \right)_{pT} = \mu \]

\[ \left( \frac{\partial G}{\partial T} \right)_{N,p} = -\frac{1}{\lambda^3 Y_N} \int dV v e^{-\beta p v - \beta F} \left( \frac{p v + F}{T} + \frac{\partial F}{\partial (\omega T)} \right) \]

\[ -G_T = 0 \]

\[ \left( \frac{\partial G}{\partial T} \right)_{N,p} = \langle S' \rangle \]

b) Expand \(( -\beta p v - \beta F(\nu, \omega, T) )\) about maximum at \( v = \bar{v} \)

\[ -\beta p v - \beta F = -\beta \bar{v} - \beta F(\nu, \bar{v}, T) - \beta \frac{\partial F}{\partial (\omega T)} \bigg|_{v=\bar{v}} (v-\bar{v})^2 \]

where \(-\beta p - \beta \frac{\partial F}{\partial (\omega T)} \bigg|_{v=\bar{v}} = 0\) i.e. \( p \) is the pressure.
\[-\beta \frac{\partial^2 F}{\partial V^2} = -\beta \left( \frac{\partial P}{\partial V} \right)_T \frac{1}{V} \frac{\partial V}{\partial P_T} = \frac{-1}{\kappa V T X_T} \leq 0\]

\[Y_N(p, T) \approx \exp \left( -\beta p \bar{V} - \beta F(n, \bar{v}, T) \right) \sqrt{\frac{2\pi \kappa T X_T \bar{V}}{\lambda^3}} \exp \left( -\frac{(V - \bar{V})^2}{2\kappa T X_T \bar{V}} \right)\]

\[G = -kT \ln Y_N = F + PV + \ln \left( \sqrt{\frac{2\pi \kappa T X_T \bar{V}}{\lambda^3}} \right)\]

Ignore in thermal limit

\[Y_N(p, T) = \frac{1}{\lambda^3} \int_{N V_0}^{\infty} dV e^{-\beta p \bar{V}} \left( \frac{V - N V_0}{\lambda^3} \right)^N \frac{1}{N!}\]

\[Y_N(p, T) = \frac{1}{\lambda^3} \int_0^{\infty} dV' e^{-\beta p (V' + N V_0)} \left( \frac{V'}{\lambda^3} \right)^N \frac{1}{N!}\]

\[= \frac{P - \beta p N V_0}{\lambda^{3(N+1)} (\beta p)^N N!} \int_0^{\infty} ds e^{-s} s^N\]

\[Y_N(p, T) = \frac{\exp(-\beta p N V_0)}{(\beta p V_0)^{N+1}}\]

\[G = -kT \ln Y_N = p V_0 N + N k T \ln \left( \beta p \lambda^3 \right)\]

\[\left( \frac{\partial G}{\partial P} \right)_{V, T} = V = N V_0 + \frac{N k T}{P} \quad \Rightarrow P = \frac{N k T}{V - N V_0}\]
\[ A(N, v, \nu, T) = -kT \ln \left( \frac{1}{N!} \left( \frac{v-N\nu_0}{\lambda^3} \right)^N \right) \]

\[ A(N, v, \nu, T) = -NkT \ln \left( \frac{v-N\nu_0}{N\lambda^3} \right) - NkT \]

\[ P = \left( \frac{\partial A}{\partial v} \right)_{N, T} = \frac{NkT}{v-N\nu_0} \text{ same as from Gibbs.} \]

\[ G = A + PV = -NkT \ln \left( \frac{v-N\nu_0}{N\lambda^3} \right) - NkT + PV \]

\[ V = N\nu_0 + \frac{NkT}{P} \quad \text{so} \]

\[ G = -NkT \ln \left( \frac{NkT}{P\lambda^3} \right) - NkT + P \left( N\nu_0 + \frac{NkT}{P} \right) \]

\[ G = NkT \ln (\beta P\lambda^3) + N\rho \nu_0 \]
\[ L = \frac{1}{2} m (\dot{\theta}^2 + \dot{\phi}^2) + \frac{1}{2} m (L^2 \sin^2 \phi) + mgL \cos(\phi) \]

\[ P_\theta = \frac{\partial L}{\partial \dot{\theta}} = mL^2 \dot{\theta} \]

\[ P_\phi = \frac{\partial L}{\partial \dot{\phi}} = mL^2 \dot{\phi} \sin^2 \phi \]

\[ H = \dot{\theta} P_\theta + \dot{\phi} P_\phi - L \]

\[ H = \frac{\partial^2 P_\theta^2}{2mL^2} + \frac{P_\phi^2}{2mL^2 \sin^2 \theta} - mgL \cos \theta \]

\[ Q_1 = \frac{1}{h} \int dp_\theta \int dp_\phi \int d\theta \int d\phi \exp(-\beta H) \]

\[ = \frac{1}{\hbar} \sqrt{\frac{2\pi m L^2 kT}{h^2}} \]

\[ = \frac{1}{\hbar} \int d\theta \int d\phi \exp \left( + \frac{\beta mgL \cos \theta}{\sqrt{2\pi mL^2 kT}} \frac{1}{\hbar} \sqrt{2\pi mL^2 kT} \right) \]

\[ = \left( \frac{2\pi mL^2 kT}{h^2} \right)^{1/2} \int d\theta \int d\phi \exp \left( \frac{\beta mgL \cos \theta}{\sqrt{2\pi mL^2 kT}} \right) \]

\[ = \left( \frac{2\pi mL^2 kT}{h^2} \right)^{1/2} \int \frac{d(\cos \theta)}{\sqrt{2\pi mL^2 kT}} \exp \left( \frac{\beta mgL \cos \theta}{\sqrt{2\pi mL^2 kT}} \right) \]

\[ Q_1 = \frac{2\pi}{h^2} \left( \frac{2\pi mL^2 kT}{h^2} \right) \left( \frac{kT}{mgL} \right) 2 \sinh \left( \frac{\beta mgL}{kT} \right) \]

\[ A = -kT \ln \left( \frac{T_m (2\pi mL^2)}{h^2 \beta} \right) \frac{\sinh(\beta mgL)}{\beta mgL} \]

\[ U = +\frac{\partial A}{\partial \beta} = 2kT - \frac{mgL \cos \phi}{\sinh(\beta mgL)} \]

\[ \beta kT \ll mgL \quad U \approx 2kT - mgL \quad \text{equipartition: } \]

\[ \beta \text{ kT} \ll mgL \quad u \approx 2kT - mgL \quad 4 \text{ quanta per degree of freedom} \]
For $kT \gg mg L$,

$$U \approx 2kT - \frac{mgL}{\beta mg L} = kT$$

**Two quadratic degrees of freedom**

*Have lost* the $x$-$y$ coordinate $\frac{1}{2}(mgx^2 + y^2)$ terms.

Since $kT \gg mgL$,

not oscillating about equal point.

---

\[
\left( \frac{\partial^2 U}{\partial \phi^2} \right) = C
\]

\[
1 \quad \text{to} \quad kT \quad \frac{mgL}{mgL}
\]