1. (10%) Assume we move one mirror of a Michelson interferometer through a distance of $3.142 \times 10^{-4}$ m and we see 850 bright fringes pass by. What is the illuminating wavelength?

2. (10%) Looking into the Michelson interferometer, we see a dark central disk surrounded by concentric light and dark rings. One mirror is 2 cm farther from the beam splitter than the other, and $\lambda = 500$ nm. What is the order of the central disk and the 6th dark ring?

3. (15%) If we coat glass ($n = 1.5$) with another material ($n = 2.0$), what thicknesses give maximum reflection? Minimum reflection? Assume that $\lambda = 500$ nm.

4. (15%) A Michelson interferometer can be used to determine the index of refraction of a gas. The gas is made to flow into an evacuated glass cell of length $l$ placed in one arm of the interferometer. The interference fringes are counted as they move across the view aperture when the gas flows into the cell. Show that the effective optical path difference of the light beam for the full cell versus the evacuated cell is $\frac{1}{2}nl$, where $n$ is the index of refraction of the gas, and hence that a number $\frac{\lambda}{4(1-n)}N$ fringes move across the field of view as the cell is filled. How many fringes would be counted if the gas were air ($n = 1.0003$) for a 10-cm cell using yellow sodium light $\lambda = 590$ nm?

5. (15%) In a two-slit Young interference, the aperture-to-screen distance is 2m and the wavelength is 600 nm. If it is desired to have a fringe spacing of 1 mm, what is the required slit separation? If a thin plate of glass ($n=1.5$) of thickness 0.05 mm is placed over one of the slits. What is the resulting lateral fringed displacement at the screen?

6. (15%) A metal ring is dipped into a soapy solution ($n = 1.34$) and held in a vertical plane so that a wedge-shaped film formed under the influence of gravity. At near-normal illumination with blue-green light ($\lambda = 488$ nm) from an argon laser, one can see 12 fringes per cm. Determine the wedge angle of the soap film.

7. (20%) Write a computer program to add 7 harmonic waves together graphically. These waves have the same wavelength and amplitude but each differs in phase from the next by 20°. (b) Write a computer program to show graphically that for what value of the phase difference the resultant wave would have zero amplitude assuming equal phase difference between each wave and its neighbor.
EECS 4800 HW #4 Solution

1. The phase difference is
\[
\delta = 2d \frac{2\pi}{\lambda_0}
\]

And there are 850 bright fringes \( \Rightarrow m = 850 \)

Therefore, the total phase difference will be
\[
\delta = 2\pi m
\]

\[
2d \frac{2\pi}{\lambda_0} = 2\pi m
\]

\[
\lambda_0 = \frac{2d}{m} = \frac{2(3,142 \times 10^{-4})}{850} = 739 \text{ nm}
\]

For the center disk
\[
\delta = \frac{2\pi}{\lambda} \Rightarrow 2d = 2\pi m
\]

\[
m = \frac{2d}{\lambda} = \frac{2(0.02)}{500 \times 10^{-9}} = 80,000
\]

The order is 80,000 for the center ring.

For the sixth dark ring
\[
m = 80,000 - 5 = 79,995
\]
the phase shift due to material is
\[ \delta = \frac{2\pi}{\lambda_0} \left( 2n_2 d \right) \]

For the first interface \( n_{air} < n_2 \Rightarrow \pi \) phase shift
second \( n_2 > n_{glass} \Rightarrow 0 \) phase shift

the total phase shift
\[ \delta_t = \delta + \pi \]
\[ = \frac{4\pi}{\lambda_0} n_2 d + \pi \]

In order to have maximum reflection, the two reflection have to be constructive interference
\[ \delta_t = \frac{4\pi}{\lambda_0} n_2 d + \pi = 2\pi m, \quad m = 0, 1, 2, \ldots \]
\[ \Rightarrow d = (2m-1) \frac{\lambda_0}{4n_2} \quad m = 1, 2, \ldots \]
or \[ d = (2m+1) \frac{\lambda_0}{4n_2} \quad m = 0, 1, 2, \ldots \]
\[ = (2m+1) \left( \frac{500 \text{ nm}}{4(2)} \right) \]
\[ = (2m+1) (62.5 \text{ nm}) \quad m = 0, 1, 2, \ldots \]

To have minimum reflection, \( \Rightarrow \) destructive interference
\[ \delta_t = 4\pi n_2 d / \lambda_0 + \pi = (2m+1)\pi, \quad m = 0, 1, 2, \ldots \]
\[ \Rightarrow d = 2m \frac{\lambda_0}{4n_2} \quad m = 1, 2, \ldots \]
\[ = 2m (62.5 \text{ nm}) \quad m = 1, 2, \ldots \]
The optical path length for an empty cell 
\[(OPL)_{\text{empty}} = 2l\]

full cell 
\[(OPL)_{\text{full}} = 2nl\]

\[\Rightarrow \text{the optical path difference}\]
\[\text{OPD} = (OPL)_{\text{full}} - (OPL)_{\text{empty}}\]
\[= 2nl - 2l\]
\[= 2(n-1)l\]

the number of fringes \(N\) (including bright and dark fringes)
\[N = 2 \frac{2(n-1)l}{\lambda} = 4(n-1) \frac{l}{\lambda}\]

therefore, if \(n = 1.0003\), \(l = 0.1\) m \(\lambda = 590 \times 10^{-9}\) m

\[N = \frac{4(1.0003 - 1)(0.1)}{590 \times 10^{-9}} = 203\]
5. a) For the Young interference

\[ \Delta x = \frac{\lambda D}{h} \]

\[ \Rightarrow h = \frac{\lambda D}{\Delta x} = \frac{(600 \times 10^{-9}) \times (2)}{0.001} = 1.2 \text{ mm} \]

b) the OPD of the two slits with the material is

\[ S = \frac{2\pi}{\lambda} \cdot \frac{h x}{D} \]

Now, the extra OPD induced by the material is

\[ S' = n \frac{2\pi}{\lambda} \cdot d \]

In order to cancel this phase difference

\[ \frac{2\pi}{\lambda} \cdot \frac{h x}{D} = n \frac{2\pi}{\lambda} \cdot d \]

\[ x = \frac{ndD}{h} = \frac{(1.5)(0.05 \times 10^{-3}) \times (2)}{(1.2 \times 10^{-3})} = 12.5 \text{ cm} \]
# of fringes/cm = 12 = \frac{0.01}{\Delta x}

\Rightarrow \Delta x = 8.3 \times 10^{-4} \text{ m}

the OPD for the two waves reflected on the 1st and 2nd surfaces,

$$\text{OPD}_m = 2n \, dm$$

$$= 2n \, x_m \sin \alpha$$

$$= 2n \, x_m \, \alpha$$ for small \( \alpha \)

for adjacent fringes, the difference of the OPD should equal to one optical wavelength

$$\Delta \text{OPD} = 2n(\Delta x) \, \alpha = \lambda$$

$$\Rightarrow \lambda = \frac{\text{488} \times 10^{-9}}{2n \, \Delta x} \approx \frac{488 \times 10^{-9}}{2(1.34)(8.3 \times 10^{-4})}$$

$$= 2.2 \times 10^{-4} \text{ rad or 0.013°}$$
% Problem #7 Part 1

clear all;

wt=0:.1:4*pi;
phase=20/180*pi;

E1=cos(wt+0*phase);
E2=cos(wt+1*phase);
E3=cos(wt+2*phase);
E4=cos(wt+3*phase);
E5=cos(wt+4*phase);
E6=cos(wt+5*phase);
E7=cos(wt+6*phase);

Etotal=E1+E2+E3+E4+E5+E6+E7;

plot(wt',[Etotal' E1' E2' E3' E4' E5' E6' E7']);
% Problem #7 part 2
% phase = 360/7=51.43

clear all;

wt=0:.1:4*pi;

phase=51.43/180*pi;

E1=cos(wt+0*phase);
E2=cos(wt+1*phase);
E3=cos(wt+2*phase);
E4=cos(wt+3*phase);
E5=cos(wt+4*phase);
E6=cos(wt+5*phase);
E7=cos(wt+6*phase);

Etotal=E1+E2+E3+E4+E5+E6+E7;

plot(wt',[Etotal' E1' E2' E3' E4' E5' E6' E7']);