Homework 9

Phys4410, Spring 2013

Due Apr 12, 2013

1) 9.12

2) A hydrogen atom in its ground state \((n, l, m) = (1, 0, 0)\) is placed between the plates of a capacitor. A time dependent but spatially uniform electric field is created

\[
E = 0 \text{ for } t < 0 \\
E = E_0 \hat{z} e^{-t/\tau} \text{ for } t > 0.
\]

Using first order time dependent perturbation theory calculate the probability that the atom is found in one of the \(2p\) or in the \(2s\) states at time \(t \gg \tau\). Discuss all 3 \(2p\) states. You don’t need to evaluate the radial integral, but perform all other (angular) integrals.

3) A harmonic oscillator is described by the Hamiltonian

\[
H = \frac{p^2}{2m} + \frac{1}{2} m \omega(t)^2 x^2
\]

where \(\omega(t) = \omega_0 + \delta \cos(ft)\) and \(\delta \ll \omega_0\). At time \(t = 0\) the oscillator is in its ground state.

   a) Use first order perturbation theory to find the rate for the particle to be found in any excited state at a time \(t \gg 0\).

   b) Use first order perturbation theory to find the probability that the particle is found in any excited state at a time \(t > 0\). You will get a fairly complicated expression - interpret it, for example, by plotting the transition rate as the function of \(f\) at fixed time. (Choose some simple value for \(\omega_0\), like \(\omega_0 = 1\).) Consider small and large \(t\) values.

4) The Hamiltonian of a charged particle in an external electromagnetic field is

\[
H = \frac{1}{2m} (\vec{p} - \frac{e}{c} \vec{A})^2 + e \Phi + V(r)
\]

a) Show that a constant magnetic field can be described by the vector potential \(\vec{A} = -\frac{1}{2} \vec{r} \times \vec{B}\).

b) Show that the Hamiltonian in a constant magnetic field \(\vec{B} = B\hat{z}\) is

\[
H = \frac{1}{2} \vec{p}^2 + \frac{e}{2mc} \vec{B} \vec{L} + \frac{e^2}{8mc^2} B^2 (x^2 + y^2)
\]
where $\vec{L} = \vec{r} \times \vec{p}$ is the angular momentum operator.

c) When discussing the Zeeman effect we intuitively added the second term as interaction Hamiltonian. Are we justified neglecting the last term? Why?