1) Two spin 1/2 particles are bound in an s-wave \((l=0)\) state. An external magnetic field is applied and the system is described by the Hamiltonian

\[ H = \epsilon \vec{S}_1 \cdot \vec{S}_2 + b_1 S_{1z} - b_2 S_{2z} \]

Assume that \(b \ll \epsilon\).

a) What is the degeneracy of \(H_0 = \epsilon \vec{S}_1 \cdot \vec{S}_2\)? Find the correct 0th order eigenkets and first order energy corrections for the perturbed Hamiltonian \(H\). How is the degeneracy of the ground state lifted by this perturbation? Make a sketch.

b) Find the first order eigenkets and second order energy corrections.

2) Griffith 6.32

3) Griffith 6.29

4) Griffith 6.30

5) Consider the addition of two angular momenta, one is \(l_1 = 1\), the other is \(l_2\), an arbitrary integer.
   a) What are the possible values of the total angular momentum \(J = l_1 + l_2\)?
   b) The eigenkets \(|j, m>\) can be expressed as a linear combination of the eigenkets \(|l_1, m_1, l_2, m_2>\). Write an expression for this. You don’t have to evaluate the coefficients, just write an expression in terms of Clebsh-Gordon coefficients. What is the relation between \(m_1\), \(m_2\) and \(m\) for non-zero Clebsh-Gordon coefficients?
   c) The eigenkets \(|j, m>, |l_1, m_1>, \text{ and } |l_2, m_2>\) can be represented in terms of the spherical harmonics \(Y_{l,m}(\theta, \phi)\). What is the representation of the ket \(|l_1, m_1, l_2, m_2>\)?
   d) Now evaluate the inner product \(<j, m|l_1, m_1, l_2, m_2>\) in terms of Clebsh-Gordon coefficients. Express it also as an explicit integral of the spherical harmonics - this gives you an easy way to evaluate integrals of three spherical harmonics.
   e) Apply the result of d) to the special cases of \(l_2 = 0\) and \(l_2 = 1\). Specify which integrals
are non-zero and find their explicit value using a table of Clebsch-Gordon coefficients.

f) The spherical harmonic \( Y_{1,0} = \sqrt{3/4/\pi} \cos(\theta) \). Apply the result of e) to evaluate the matrix elements \( < l = 1, m | \cos(\theta) | l = 1, m' > \) and \( < l = 1, m | \cos(\theta) | l = 0, m' > \). These were the matrix elements you needed for Problem 6.36b. Check that you get the same results.

g) Consider now the general case \( < l, m | \cos(\theta) | l', m' > \). What constraints must \( l, m, l', m' \) satisfy for this matrix element to be non-zero?