Homework 2

(Due Date: Wed, Jan 28, in class. Recall: late homework will not be accepted)

NOTE: Be sure to show your work and explain what you are doing

1. Consider a system composed of two spin 1/2 particles whose orbital variables are ignored, with Hamiltonian
   \( \hat{H} = \omega_1 \hat{S}_1z + \omega_2 \hat{S}_2z \). Suppose the initial state of the system at time \( t = 0 \) is
   \( |\psi(0)\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} \). If \( \hat{S}^2 = (\hat{S}_1 + \hat{S}_2)^2 \) is measured at time \( t \), what results can be found and with what probabilities? (30 pts)

2. A system of three nonidentical particles of spin 1/2 whose spin operators are \( \hat{S}_1, \hat{S}_2, \hat{S}_3 \) is governed by the Hamiltonian
   \[
   \hat{H} = A\hat{S}_1 \cdot \hat{S}_2 + B(\hat{S}_1 + \hat{S}_2) \cdot \hat{S}_3
   \]
   Find the energy levels and their quantum numbers and degeneracies. (30 pts)

3. The total angular momentum of the hydrogen atom is \( \hat{F} = \hat{J} + \hat{I} \), where \( \hat{I} \) is the nuclear spin and \( \hat{J} = \hat{L} + \hat{S} \) is the total angular momentum. The eigenvalues of \( \hat{L}^2, \hat{J}^2, \hat{S}^2, \hat{I}^2 \) and \( \hat{F}^2 \) are \( \hbar^2 l(l+1) \), \( \hbar^2 J(J+1) \), \( \hbar^2 S(S+1) \), \( \hbar^2 I(I+1) \) and \( \hbar^2 F(F+1) \) respectively. For the hydrogen atom, \( I = S = 1/2 \). Assuming that the atom is in an fixed \( nl \) state, with \( n = 3 \) and \( l = 2 \) (30 pts: 15+15)
   
   (a) What are the possible values of the quantum number \( J \) and the corresponding degeneracies
   
   (b) What are the possible values of the quantum number \( F \) and the corresponding degeneracies