NOTE: Be sure to show your work and explain what you are doing

1. [45 points: 15 each]

We derived in class that if $V(t) = V_0 \cos(\omega t)$, the Fermi-Golden rule formula for the transition probability per unit time is

$$\Gamma_{\beta \rightarrow \alpha} = \frac{\pi}{2\hbar} |\langle \beta | V_0 | \alpha \rangle|^2 [\delta(E_\beta - E_\alpha - \hbar \omega) + \delta(E_\beta - E_\alpha + \hbar \omega)]$$

Let’s consider again the hydrogen atom in the $F = 0$ state (where $\vec{F} = \vec{S}_{\text{proton}} + \vec{S}_{\text{electron}}$) in a magnetic field $\vec{B} = \vec{B}_0 \cos(\omega t)$. The B-field induces a $V = \frac{e}{m_e} \vec{B} \cdot \vec{S}_{\text{electron}}$ (the $\frac{e}{m_p} \vec{B} \cdot \vec{S}_{\text{proton}}$ is too small to matter).

(a) Compute the transition probability per unit time, $\Gamma$, using the Fermi Golden rule to all possible $F$ states if $\vec{B}_0 = \hat{z}B_0$.

(b) The B-field is part of an electromagnetic field with energy density per frequency interval $\rho(\omega)$. Integrate the $\Gamma$ you got in a) to find the rate at which atoms are excited to the $F = 1$ state.

(c) If $\vec{B}_0 = \hat{z}B_0$ what transitions are allowed? How do the rate of these transitions compare to the ones found in b).

2. [45 points: 15 each] We have argued that the spontaneous emission rate for an atom is given by

$$\gamma_{\beta \rightarrow \alpha} = \frac{|d_{\beta \alpha}|^2 (\omega c)}{3\pi \hbar \epsilon_0 }$$

(a) Compute the dipole matrix element for the $2p \rightarrow 1s$ transition in hydrogen. Is there one number to compute here, or several? If several, compute them all.

(b) What is the lifetime, in seconds, of the 2p state?

(c) Suppose we include the electron spin in the hydrogen 2p lifetime calculation, i.e., consider the eight $2p_{3/2} \rightarrow 1s_{1/2}$ transitions. Which of these transitions is allowed by electric dipole radiation? What are their relative spontaneous decay rates? (You may ignore the fine structure splitting energy as compared to the 1s-2p energy difference.)