1) (30 points) In class we described the \( \text{H}_2^+ \) molecular ion using the variational trial wave function
\[
\psi_g = A[\psi_{100}(r_1) + \psi_{100}(r_2)],
\]
where \( \psi_{100} \) is a plain hydrogenic ground state wave function, and \( r_1 \) and \( r_2 \) represent the distance from the electron to proton 1 and proton 2, respectively. Minimizing the electronic energy for fixed distance \( R \) between the protons, we arrived at an estimate for the electronic energy,
\[
V_{el,g}(R) = \langle \psi_g | H_{el} | \psi_g \rangle,
\]
for each value of \( R \).

a) Make a plot of \( V_{el,g}(R), V_{pp}(R) \), and \( V_g(R) = V_{el,g}(R) + V_{pp}(R) \), where \( V_{pp}(R) \) is the proton-proton repulsion. The result, \( V_g(R) \), is the potential acting between the protons. (Note: here and in the following you should make a careful plot, not a freehand drawing). The minimum of this potential, relative to its value when \( R \rightarrow \infty \), is an approximation to the \( \text{H}_2^+ \) binding energy. What is this binding energy?

b) Another perfectly good electronic wave function is given by
\[
\psi_u = A[\psi_{100}(r_1) - \psi_{100}(r_2)].
\]
Calculate the electronic energy \( V_{el,u}(R) = \langle \psi_u | H_{el} | \psi_u \rangle \) for this state. (Hint: you should be able to use the same integrals that Griffiths already works out for the other case). Make a similar plot as in part a), i.e., plot \( V_{el,u}(R), V_{pp}(R) \), and
\[
V_u(R) = V_{el,u}(R) + V_{pp}(R). \]
Do the protons stick together when the electron is in this state?

c) Based on your results above, and neglecting the Coulomb interaction between electrons, what is the binding energy of the neutral \( \text{H}_2 \) molecule in its ground state? This molecule consists of two protons and two electrons.

2) (10 points) Rederive the WKB expression for a wave function in one dimension, in the classically allowed region. Only this time, use a sine function rather than the exponential function we used in class.