Physics 4410 Homework #12
Due Wednesday, Dec. 3, IN CLASS. Recall: late homework will not be accepted.
Be sure to show your work and explain what you are doing.

Reminder: the second midterm exam will be held in class, on Nov. 21.

1) (10 points) Consider strong, near-resonant light driving a two-level system. Make a plot of the excited state probability versus time, assuming the system started in its ground state at time \( t = 0 \). Make three curves on the same plot, for the detunings
   a) \( \delta = 0 \)
   b) \( \delta = \Omega \), where \( \Omega \) is the Rabi frequency
   c) \( \delta = 3\Omega \)

2) (15 points) In a resonant \( \pi \)-pulse, you set the detuning \( \delta = 0 \) and apply a light pulse for a certain duration \( T \) such that the atomic population is entirely transferred to the excited state.
   a) How long is \( T \), in terms of the Rabi frequency \( \Omega \)?
   b) After two \( \pi \)-pulses are applied, the population returns to the ground state. However, the probability amplitude of the ground state wave function has acquired a phase factor. What is this phase factor? In other words, what is the phase of \( c_a(2T) \), relative to \( c_a(0) \)?
   c) What is a \( \pi/2 \) pulse? What state are the atoms left in after such a pulse, if they began in the ground state?

Remark: This ability to create superposition states is useful in applications to quantum information.

3) (15 points) In formal scattering theory, it turns out it’s really useful to be able to solve a Schrodinger-like equation for a 3D delta function potential. To this end, show that
   \[
   \psi(\vec{r}) = \frac{1}{4\pi} \frac{e^{i k r}}{r}
   \]
   is a solution to the equation
   \[
   -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{2m}{\hbar^2} \delta(\vec{r}) = E \psi
   \]
   where, as usual, \( k^2 = 2mE/\hbar^2 \). (This is not a typo: the delta function is not multiplied by the wave function here. So this is a Schrodinger-like equation, not exactly a Schrodinger equation.)
   Hint: along the way, you will need to prove that \( \nabla^2 (1/r) = -4\pi \delta(\vec{r}) \).