Physics 4410 Homework #11
Due Wednesday, Nov. 12, IN CLASS. Recall: late homework will not be accepted.
Be sure to show your work and explain what you are doing.

1) (15 points) By a long, winding process, we have argued that the spontaneous emission rate for an atom is given by

$$\gamma = \frac{|d_{ba}|^2 (\omega_0)^3}{3\pi\hbar c}$$

a) Compute the dipole matrix element for the $2p \rightarrow 1s$ transition in hydrogen. Is there one number to compute here, or several? If several, compute them all.
b) What is the lifetime, in seconds, of the $2p$ state? (Note, lifetime is defined as the “1/e” lifetime.)

2) (15 points) Suppose we include the electron’s spin in the hydrogen $2p$ lifetime calculation, i.e., consider the eight $2p_{3/2} \rightarrow 1s_{1/2}$ transitions and the four $2p_{1/2} \rightarrow 1s_{1/2}$ transitions.
a) Which of these transitions is allowed by electric dipole radiation?
b) What are the relative spontaneous decay rates of the allowed transitions? (You may ignore the fine structure splitting energy as compared to the $1s$-$2p$ energy difference.)

Remark: This exercise shows that, interestingly, the spin state of an atom can affect its lifetime, even if the light acts only on the electron’s orbital degrees of freedom.

3) (10 points) Another kind of “perturbation” is one in which the Hamiltonian changes instantaneously into a different Hamiltonian, i.e., we have

$$H = \begin{cases} H_1, & t < 0 \\ H_2, & t \geq 0 \end{cases}$$

For two different Hamiltonians $H_1$ and $H_2$, both of which are independent of time.
(Example: you have an unperturbed atom, and then suddenly turn on a constant electric field.) Let $|\phi_n^1\rangle$ be the energy eigenstates of $H_1$, and $|\phi_m^2\rangle$ be the energy eigenstates of $H_2$. Suppose your atom starts in the state $|\phi_n^1\rangle$. What is the probability that at $t > 0$, your atom is in the state $|\phi_m^2\rangle$ of the new Hamiltonian?