Physics 4410 Homework #1
Due Wednesday, Sept. 3, IN CLASS. Recall: late homework will not be accepted.
Be sure to show your work and explain what you are doing.

1) (10 points) Return to the analogy made in class, between a spin-1/2 system and the polarization of a beam of light.
   a) Suppose light is polarized by passing it through a linear polarizer that is vertically oriented. Call the intensity of this polarized light $I$. Now pass this light through a linear polarizer that is rotated $45^\circ$ with respect to the vertical. What is the intensity of the light that passes through the second polarizer, in terms of $I$?
   b) Suppose you pass some spin-1/2 atoms through a Stern-Gerlach (SG) apparatus, and select those atoms that are spin-up according to a $z$-axis in the laboratory. Call the flux of this beam $F$. You then pass this beam through a second SG apparatus whose axis is along the $x$-axis in the laboratory. If you select the atoms that are spin-up relative to the $x$-axis, what is the flux of this beam, in terms of $F$?

2) (10 points) Suppose you have a Stern-Gerlach apparatus that is initially oriented along the $z$-axis, so that a beam of incident atoms is separated into two distinct beams with the quantum numbers $|\uparrow>_z$ and $|\downarrow>_z$, representing the usual spin-up and spin-down with respect to the $z$-axis. Now rotate the apparatus in the $x$-$z$ plane so that it makes an angle $\theta$ with respect to the $z$-axis.
   a) Write down the spin operator that describes this apparatus. In the basis of states $|\uparrow>_z$ and $|\downarrow>_z$, what is the matrix of this operator?
   b) What are the eigenstates of the operator in part a? Show that they reduce to $|\uparrow>_z$ and $|\downarrow>_z$ in the limit $\theta \to 0$. (Hint: L’Hospital’s rule for taking limits may be useful).

3) (20 points) A spin-1/2 atom is initially in an eigenstate of $S_x$ with eigenvalue $+\hbar/2$ at time $t = 0$. At that time a field $\vec{B} = B\hat{z}$ is turned on, after which the spin precesses for a time $T$. At $t = T$ the field is suddenly rotated to $\vec{B} = B\hat{y}$ and the atom is allowed to precess for an additional time $T$. After all this, at time $t = 2T$, a measurement is made of $S_x$. What is the probability that the measured value is $+\hbar/2$, i.e., right back where the spin started?