1. Kittel Ch. 2, Problem 2. Parts b and c only. In part c, also describe and sketch the Wigner-Seitz cell of the hexagonal position space lattice. (Note: the first Brillouin zone is nothing but the Wigner-Seitz cell of the reciprocal lattice.)

2. The diamond lattice can be described as a FCC position space Bravais lattice with a two-site basis. Use the primitive vectors

\[ \vec{a}_1 = \frac{a}{2} (\hat{y} + \hat{z}) \]  
\[ \vec{a}_2 = \frac{a}{2} (\hat{z} + \hat{x}) \]  
\[ \vec{a}_3 = \frac{a}{2} (\hat{x} + \hat{y}) , \]

and use the basis vectors

\[ \vec{c}_1 = 0 \]  
\[ \vec{c}_2 = \frac{a}{4} (\hat{x} + \hat{y} + \hat{z}) . \]

(a) Find primitive vectors for the reciprocal lattice. Show that the reciprocal lattice is BCC, and that every reciprocal lattice vector is either of the form

\[ \vec{Q} = \frac{4\pi}{a} [n_x \hat{x} + n_y \hat{y} + n_z \hat{z}], \]

for arbitrary integers \( n_x, n_y, n_z \), or of the form

\[ \vec{\tilde{Q}} = \frac{4\pi}{a} [(n_x + 1/2) \hat{x} + (n_y + 1/2) \hat{y} + (n_z + 1/2) \hat{z}] . \]

The reciprocal lattice points in Eq. (6) form a simple cubic lattice, and the points in Eq. (7) form another simple cubic lattice offset from the first – so, as for any BCC lattice, we can think of the reciprocal lattice as two interpenetrating simple cubic lattices.

(b) Calculate \( S_{\vec{Q}} \), the structure factor for the diamond lattice. (The diamond lattice is monatomic, so just ignore the atomic form factor by setting it equal to unity.) Show that \( S_{\vec{Q}} = 0 \) or \( 2 \) for \( \vec{Q} \)'s of the form Eq. (6), and that \( S_{\vec{Q}} = (1 \pm i) \) for \( \vec{\tilde{Q}} \)'s of the form Eq. (7). Among those \( \vec{\tilde{Q}} \)'s of the form Eq. (6), for which \( \vec{\tilde{Q}} \)'s does \( S_{\vec{\tilde{Q}}} = 0 \), and for which \( \vec{\tilde{Q}} \)'s does \( S_{\vec{\tilde{Q}}} = 2 \)?

(c) Based on the results above, how could you use x-ray diffraction to tell the difference between a monatomic solid with FCC structure, and a monatomic solid with diamond structure?

(d) Now suppose that the two basis vectors give the positions of two different atoms. That is, suppose \( \vec{c}_1 \) gives the position of atom 1, and \( \vec{c}_2 \) gives the position of atom 2, with atomic form factors \( f_1 \) and \( f_2 \), respectively. This is no longer the diamond structure but is now called the zincblende structure – an important example of a compound with this structure is GaAs. Recalculate the structure factor \( S_{\vec{Q}} \), now incorporating the atomic form factors. How do your results change as compared to the monatomic diamond case in part (b)?