1. For each of the following structures, determine whether or not it is a Bravais lattice. If it is, give the primitive vectors (this shows that it is a Bravais lattice). If not, show that the structure is not a Bravais lattice, and describe it as a Bravais lattice with a basis (to do that, you will need to give Bravais lattice primitive vectors, and a set of basis vectors).

(a) Take the two-dimensional square lattice, and add one new point at the center of each edge connecting neighboring square lattice points. After adding these extra points, remove the original lattice points, leaving only the new ones.

(b) Same as (a), but do not remove the original points – so the structure consists of the original square lattice points, plus the new points added at the center of each edge. We could call this an “edge centered square lattice.”

(c) Now start with a three-dimensional simple cubic lattice. Instead of adding an extra point to the center of every square face (to get an FCC lattice), add an extra point only to the centers of horizontal square faces (i.e. those square faces with normal vector in the z-direction). Keep all the original points, too.

2. For the two-dimensional square lattice, show that there are an infinite number of possible choices of Bravais lattice primitive vectors.

3. In class we talked about the honeycomb lattice of graphene as an example of a lattice with a (two-site) basis. Find a set of primitive Bravais lattice vectors and basis vectors describing the graphene structure, denoting the distance between nearest-neighbor C atoms by a.

4. In class we said that the sodium chloride structure can be viewed as an FCC lattice with a two-site basis. Supposing the Na-Cl distance is a, write down a set of Bravais lattice primitive vectors and basis vectors describing the sodium chloride structure.

5. (a) The BCC Bravais lattice can be viewed as a simple cubic lattice with a two-site basis. If the simple cubic Bravais lattice primitive vectors are \( \vec{a}_1 = a \hat{x} \), \( \vec{a}_2 = a \hat{y} \), \( \vec{a}_3 = a \hat{z} \), what are the basis vectors needed to describe the BCC structure?

(b) The FCC Bravais lattice can be viewed as a simple cubic lattice with a four-site basis. If the simple cubic Bravais lattice primitive vectors are \( \vec{a}_1 = a \hat{x} \), \( \vec{a}_2 = a \hat{y} \), \( \vec{a}_3 = a \hat{z} \), what are the basis vectors needed to describe the FCC structure?