Heat Engines

\[ \Delta S_{\text{sys}} = \frac{Q}{T} \]

Recall origin of \( \Delta S = \frac{Q}{T} \):

Definition of \( T \):
\[ \frac{1}{T} = \left. \frac{\partial S}{\partial u} \right|_{N,V} \Rightarrow \Delta S = \left. \frac{\Delta u}{T} \right|_{V,N} \]

At \( \text{const } V, \ W = 0 \), \( \Delta u = Q \Rightarrow \Delta S = \frac{Q}{T} \)

Even if \( V \neq \text{const} \), if process is quasi-static so that \( W = -p \, dV \), then still true that \( \Delta S = \frac{Q}{T} \)

Why? \( \Delta u = Q + W \) (1st Law)

\[ dU = T \, dS - p \, dV \]

follows from \( dS = \left. \frac{\partial S}{\partial u} \right|_{N,V} \, du + \left. \frac{\partial S}{\partial V} \right|_{u,N} \, dV \)

plus def'n of \( T \) and \( p \):

\[ \frac{1}{T} = \left. \frac{\partial S}{\partial u} \right|_{N,V}, \quad p = T \left( \frac{\partial S}{\partial V} \right)_{u,N} \]
\[ \Delta U = Q + W \]

\[ dU = Tds - pdV \]

if process is quasi-static, \( W = -pdV \)

(if not quasi-static, \( P_{\text{from outside}} \neq P_{\text{from inside}} \)

\[ \Rightarrow p \text{ not well defined} \]

\[ W = -pdV \Rightarrow Q = Tds \Rightarrow \]

\[ \Delta S = \frac{Q}{T} \]

(for any quasi-static process)

Note: always possible to create new entropy w/ non-reversible process, such as free expansion. So, in general,

\[ \Delta S \geq \frac{Q}{T} \]

(= for reversible processes only)

Heat engine: device that converts heat into work.

Examples: steam turbine, internal combustion engine

Almost all our electricity comes from heat engines. Both coal-burning and nuclear power plants use steam turbines.
efficiency \( e = \frac{W}{Q_h} \)

\( Q_h = W + Q_c \) (1st law)

\[ \Rightarrow e = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \]

Per cycle, \( \Delta S_{\text{engine}} = 0 \), since returns to original state

Entropy leaving hot reservoir \( |\Delta S_h| = \frac{Q_h}{T_h} \)

Entropy entering cold reservoir \( \Delta S_c = \frac{Q_c}{T_c} \)
\[ \Delta S_{universe} = \Delta S_{hi-T} + \Delta S_{engine} + \Delta S_{low-T} \]

\[ = \frac{Q_c}{T_c} - \frac{Q_h}{T_n} \geq 0 \quad \text{2nd Law} \]

\[ \frac{Q_c}{T_c} \geq \frac{Q_h}{T_n} , \quad \frac{Q_c}{Q_h} \geq \frac{T_c}{T_n} \]

\[ \Rightarrow e = 1 - \frac{Q_c}{Q_h} \leq 1 - \frac{T_c}{T_n} \]

Note \( T_c, T_n \) are temps of reservoirs, not engines. To get high \( e \), need \( T_c \ll T_n \).

Most efficient (but least practical) engine is \textit{Carnot Cycle engine}.

No new entropy generated, every step reversible.
Refrigerator = heat engine run in reverse

\[ W, Q_h, Q_c > 0 \] (in directions shown)

1st Law: \[ W + Q_c = Q_h \] (per cycle)

Coefficient of performance,

\[ \text{COP} = \frac{\text{benefit}}{\text{cost}} = \frac{Q_c}{W} \]

\[ \text{COP} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{1}{\left(\frac{Q_h}{Q_c} - 1\right)} \]

\[ \Delta S_{\text{universe}} = + \frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geq 0 \]

2nd Law

\[ s_{\text{gained}} \leq s_{\text{lost by}} \]

by hot res. cold res.

\[ \frac{Q_h}{Q_c} \geq \frac{T_h}{T_c} \Rightarrow \text{COP} \leq \frac{1}{\left(\frac{T_h}{T_c} - 1\right)} \]

Kitchen fridge: \[ T_h \approx 293K \]
\[ T_c \approx 263K \]

\[ \text{COP} = \frac{1}{\frac{293}{263} - 1} \leq 9 \]

\[ \Rightarrow 1 \text{ J of electrical energy in} \]
\[ 9 \text{ J of heat pumped from interior} \]
\[ (9+1) \text{ J of heat pumped into room} \]