Chapter 2 Concept Tests

2.1. What is the probability of being dealt the following nice poker hand (a royal flush) A♥, K♥, Q♥, J♥, 10♥ vs the probability of being dealt this hand: 3♣, 9♣, 2♥, J♥, 10♣
A) Probability of getting this Royal Flush < prob. of this junk hand
B) Probability of getting this Royal Flush > prob. of this junk hand
C) Probability of getting this Royal Flush = prob. of this junk hand

Answer: Same probability for both.

2.2. How many ways are there to arrange the cards in 52-cards?
A) 52! B) 52^52 C) 2^52 D) Some other number

Answer: 52! = number of permutations (arrangements) of 52 objects

2.3. You flip a coin 10 times. How many sequences (like HHTTHTHTTH) or microstates are possible?
A) 10! B) 10^2 C) 2^10 D) Some other number

Answer: 2^{10} = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = (2 choices for 1^{st} coin) \times (2 choices for 2^{nd} coin) \times etc.

2.4. How many 5-card hands are possible from a 52-card deck? (Order of cards in a hand not important.)
A) 52\times51\times50\times49\times48 B) 52^5 C) \frac{52\times51\times50\times49\times48}{5\times4\times3\times2}

D) some other answer

Answer: “52 choose 5” = \binom{52}{5} = \frac{52!}{5!(52−5)!} = \frac{52\times51\times50\times49\times48}{5\times4\times3\times2}

2.5. How many 5-card ordered hands are possible?
A) 52\times51\times50\times49\times48 B) 52^5 C) \frac{52\times51\times50\times49\times48}{5\times4\times3\times2}

Answer: 52\times51\times50\times49\times48
2.6. You flip a coin 10 times. How many combinations are there with 2 heads and 8 tails? (Like TTTHTTTTHT)
A) 1024   B) 100   C) 90   D) 45   E) Some other number
Answer: “10 choose 2” = 10×9/2 = 45

2.7. You flip a coin 10 times. What is the probability that you will get 2 heads and 8 tails (in any order)?
A) 45/(2^10)   B) 45/100   C) 45/10!   D) 45/90   E) Some other number
Answer: 45/(2^10)

2.8. Einstein solid with N = 3, q = 4. What is Ω(N = 3, q = 4)?
A) 4^3   B) 3^4   C) 3•4 = 12   D) 15   E) Some other number
Answer: \( \binom{q+N-1}{q} = 15 \)

2.9. Two Einstein solids, A and B, are in thermal contact. The two solids are thermally isolated from the rest of the universe. \( N_A = N_B = 10 \). \( q_{tot} = q_A + q_B = 20 \).
How many macrostates are there?
A) 10   B) 21   C) \( 20^{20} \)   D) \( \binom{39}{20} \)   E) 20
Answer: 21, \( q_A = 0, 1, 2, .. 20 \). Any specification of \( q_A \) specifies a macrostate.

2.10. Two Einstein solids, A and B, are in thermal contact. The two solids are thermally isolated from the rest of the universe. \( N_A = N_B = 10 \). \( q_{tot} = q_A + q_B = 20 \).
Are all the microstates of solid A equally likely?
A) Yes   B) No
Answer: NO! The microstates of an isolated system are equally likely. Solid A is not isolated; it is in thermal contact with solid B.

2.11. Simplify \( \exp[N \ln(a) – \ln(b)] \)
A) Cannot be simplified.   B) \( a^N/b \)   C) \( a^N \)   D) \( N+a+b \)   E) \( a^N – b \)
Answer: \( a^N/b \)
2.12. \( q \gg N \), \( \ln \left[ q \left( 1 + \frac{N}{q} \right) \right] = ?? \)

A) \( N \) \hspace{1cm} B) \( \ln[q + (N/q)] \) \hspace{1cm} C) \( \ln q + \frac{N}{q} \)

Answer: \( \ln q + \frac{N}{q} \)

2.13. \( \delta \ll 1 \), \( \ln \left( 1 - \delta^2 N \right) = ?? \)

A) \( \ln N + \delta \) \hspace{1cm} B) \( \ln(N + \delta) \) \hspace{1cm} C) \( -N \delta^2 \)

Answer: \( -N \delta^2 \)

2.14. Consider \( \frac{(q + N)!}{q!} \) for the case \( q \gg N \).

If you wrote out all the factors, and cancelled as much as you could, how many factors would be left?
A) \( q \) factors \hspace{1cm} B) \( N \) factors \hspace{1cm} C) \( q + N \) factors \hspace{1cm} D) None of these.

Answer: \( N \) factors.

2.15. Two large Einstein solids, A and B, in thermal equilibrium.
Total nmr of energy units \( q = q_A + q_B \)
Total nmr of oscillators = \( N = N_A + N_B \)

\[ \Omega_{\text{tot,max}} = \Omega_A \cdot \tilde{q}_A \cdot \Omega_B(q - \tilde{q}_A), \]
\[ \tilde{q}_A = \text{value of } q_A \text{ which maximizes } \Omega_{\text{tot}} \]

\[ \Omega_{\text{grand}} = \sum_{q_A} \Omega_A \cdot \Omega_B(q - q_A) = \left\{ \frac{q + N - 1}{q} \right\} \]

The entropy \( S = k \ln \Omega \) of the total system (A+B) uses which \( \Omega \), in thermal equilibrium?
A) \( S = k \ln \Omega_{\text{grand}} \) \hspace{1cm} B) \( S = k \ln \Omega_{\text{tot,max}} \) \hspace{1cm} C) It doesn’t matter.

Answer: It doesn’t matter. In the thermodynamic limit (large \( N \)), the most likely macrostate is so overwhelmingly likely compared to all other macrostates that it dominates the sum in \( \Omega_{\text{grand}} \).
2.16. The entropy of a monotonic ideal gas, with N particles of mass m, in a container of volume V, is

\[ S = N k \left( \frac{5}{2} + \ln \left( \frac{V \left( \frac{4\pi m U}{N} \right)^{3/2}}{3N h^2} \right) \right) \]

Is this equivalent to

\[ S(N, V, N) = f(N) + N k \ln(V) + \frac{3}{2} N k \ln(U) \]

where f(N) is some function of N?

A) Yes, it is equivalent.  
B) No, it is not equivalent

Answer: it is equivalent

2.17. Quantum Mechanical ideal gas consisting of N particles in a box (infinite square well) of edge length L. The system is thermally isolated, so no heat allowed in or out. I slowly increase the box size L. What happens to the total energy U of the system?

A) U increases  
B) U decreases  
C) U remains constant

Answer: U decreases. Our formula for the total energy of an ideal QM gas is:

\[ U = \varepsilon \sum_{i=1}^{3N} n_i^2 = \left( \frac{h^2}{8mL^2} \right) \sum_{i=1}^{3N} n_i^2 \]

As L increases, the energy decreases, assuming that the occupation numbers remain constant. The n’s will remain constant if no heat is let into or out of the box.

2.18. The microstates of a 3D particle in a box are represented by points in “n-space”. How many microstates are there in a 5×4×3 volume of phase space?

A) Impossible to tell without more information
B) 5×4×3=60
C) Some integer (>1) multiple of 60
D) 5^{12}
E) 5^{(4+3)} = 5^7

Answer: 5×4×3=60
Moral: The number of microstates enclosed by a volume in n-space = the volume in n-space