In each problem, be sure to give the reasoning for your answer and define any variables you create. If you use a general formula, state that formula clearly before manipulating it.

1. Consider two systems A and B put in thermal contact, but isolated from any other system or environment. The figure below shows the entropy of each system, $S$, as a function of its internal energy $U$. The initial values for these systems are indicated on the figure as dots.

   (a) [3 pts] Which system is initially at higher temperature? Explain your reasoning.

   (b) [6 pts] Will the combined system come to thermal equilibrium? If so, will the final temperature be closer to the initial temperature of system A or system B? Explain your reasoning.

   (c) [6 pts] Compare the change in entropy of system A, $\Delta S_A$, to that of system B, $\Delta S_B$, and comment on whether each is positive or negative. Explain your reasoning.

2. [5 pts] A paramagnetic sample in a magnetic field ($B$) is prepared so that 40% of the magnetic dipoles ($\mu$) are in states which are anti-aligned with the direction of the magnetic field. Is the temperature of the sample positive or negative? Explain your reasoning.

3. [5 pts] The $\Delta G$ for a particular chemical reaction proposed for a new fuel cell is $-450 \text{ kJ/mole}$. This reaction would push 3 electrons around the circuit for each molecule of fuel which undergoes the reaction. Estimate the voltage of the cell ($1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$).
4. [10 pts] An Einstein solid has multiplicity \( \Omega(N, q) = \frac{(q+N-1)!}{q!(N-1)!} \) and energy \( U = q\epsilon \). From the data below, roughly estimate the chemical potential \( \mu \) in units of \( \epsilon \) at \( N = 40, \ q = 70 \). Explain your reasoning.

\[
\begin{align*}
\Omega(40, 66) &= 9.74 \times 10^{28} & \Omega(41, 66) &= 2.58 \times 10^{29} \\
\Omega(40, 67) &= 1.54 \times 10^{29} & \Omega(41, 67) &= 4.12 \times 10^{29} \\
\Omega(40, 68) &= 2.42 \times 10^{29} & \Omega(41, 68) &= 6.55 \times 10^{29} \\
\Omega(40, 69) &= 3.80 \times 10^{29} & \Omega(41, 69) &= 1.03 \times 10^{30} \\
\Omega(40, 70) &= 5.91 \times 10^{29} & \Omega(41, 70) &= 1.63 \times 10^{30} \\
\Omega(40, 71) &= 9.16 \times 10^{29} & \Omega(41, 71) &= 2.54 \times 10^{30} \\
\Omega(40, 72) &= 1.41 \times 10^{30} & \Omega(41, 72) &= 3.95 \times 10^{30}
\end{align*}
\]

5. [8 pts] Calcium carbonate, \( \text{CaCO}_3 \), has two common crystalline forms, calcite and aragonite. Thermodynamic data for these phases at 298 K and \( p = 1 \) bar are given below. Which form is stable at 298 K? Estimate the temperature (still at \( p = 1 \) bar) at which the two forms become equally stable, if such a temperature exists. Explain your reasoning.

<table>
<thead>
<tr>
<th>Substance</th>
<th>( \Delta_f H ) (kJ)</th>
<th>( \Delta_f G ) (kJ)</th>
<th>S (J/K)</th>
<th>( C_P ) (J/K)</th>
<th>V (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcite</td>
<td>-1206.9</td>
<td>-1128.8</td>
<td>92.9</td>
<td>81.88</td>
<td>36.93</td>
</tr>
<tr>
<td>Aragonite</td>
<td>-1207.1</td>
<td>-1127.8</td>
<td>88.7</td>
<td>81.25</td>
<td>34.15</td>
</tr>
</tbody>
</table>

6. [7 pts] A heat engine (not necessarily reversible) executes exactly one complete cycle. Using the usual notation defined in the diagram, write a general expression for the entropy change \( \Delta S \) of the “universe,” that is, the engine plus hot and cold reservoirs.
**Physical Constants and Formulae**

\[ k = 1.381 \times 10^{-23} \text{ J/K} \]
\[ R = 8.315 \text{ J/mol \cdot K} \]
\[ N_A = 6.022 \times 10^{23} \]
\[ h = 6.626 \times 10^{-34} \text{ J \cdot s} \]

\[ C_V = \left. \frac{\partial U}{\partial T} \right|_V \]
\[ \alpha = \frac{\Delta L/L}{\Delta T} \]
\[ B = - \frac{\Delta p}{\Delta V/V} \]
\[ \frac{1}{T} = \left. \frac{\partial S}{\partial U} \right|_{N,V} \]
\[ p = T \left. \frac{\partial S}{\partial V} \right|_{N,U} \]

\[ C_p = \left. \frac{\partial U}{\partial T} \right|_p \]
\[ \beta = \frac{\Delta V/V}{\Delta T} \]
\[ S = k \ln \Omega \]
\[ dS = \frac{C_V}{T} dT \]
\[ \mu = -T \left. \frac{\partial S}{\partial N} \right|_{U,V} \]

For adiabatic ideal gas processes: \( VT^{f/2} \) =constant and \( pV^{(f+2)/f} \) =constant.

**Multiplicities**

\[ \Omega_{\text{osc}}(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!} \]
\[ \Omega_{\text{para}}(N, n_\uparrow) = \frac{N!}{n_\uparrow!(N - n_\uparrow)!} \]
\[ \Omega_{\text{ideal gas}} = \frac{1}{N! h^{3N} (\frac{3N}{2} - 1)!} m(\sqrt{2mU})^{3N-2} \delta U \]

**Paramagnetism (\( \vec{B} \) points “up”)**

\[ U = \mu B(n_\uparrow - n_\downarrow) \]
\[ M = \mu (n_\uparrow - n_\downarrow) \]

**Sacker-Tetrode Equation**

\[ S = Nk \left[ \ln \left( \frac{V}{N} \left( \frac{4\pi mU}{3Nh^2} \right)^{3/2} \right) + \frac{5}{2} \right] \]

**Thermodynamic Potentials and Identity**

\[ H = U + pV \]
\[ F = U - TS \]
\[ G = U + pV - TS \]
\[ G = H - TS \]
\[ dU = T dS - p dV + \mu dN \]
Clausius-Clapeyron Relation

\[ \frac{dp}{dT} = \frac{\Delta S}{\Delta V} \]

van der Waals Equation

\[(p + \frac{aN^2}{V^2})(V - Nb) = NkT\]

Performance Measures

Engine Efficiency \( e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}\)

Refrigerator COP = \( \frac{Q_c}{W} = \frac{1}{Q_h/Q_c - 1}\)

Binomial Distribution

\[ P_N(n) = \frac{N!}{n!(N-n)!} p^n q^{N-n} \]

Mathematics

\[ N! \approx N^N e^{-N} \sqrt{2\pi N} \]

\[ A_d(r) = \frac{2\pi^{d/2}}{(\frac{d}{2} - 1)!} r^{d-1} \]

\[ \Gamma(n+1) = n! \]

\[ \Gamma(n+1) = n\Gamma(n) \]

\[ \Gamma\left(\frac{3}{2}\right) = \frac{1}{2}! = \frac{\sqrt{\pi}}{2} \]

\[ \sinh x = \frac{e^x - e^{-x}}{2} \]

\[ \cosh x = \frac{e^x + e^{-x}}{2} \]

\[ \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]