Physics 4230 Final Exam, Spring 2004
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This is a 2.5 hour exam. Budget your time appropriately. Good luck!

For all problems, show your reasoning clearly. In general, there will be little or no credit awarded for writing a correct answer without some indication of your reasoning.

Possibly useful information:
\[ k = 1.38 \times 10^{-28} \quad m_e = 9.1 \times 10^{-31} \quad e = 1.6 \times 10^{-19} \quad h = 6.63 \times 10^{-34} \]
(all in SI units)

1 calorie = 4.2 J

1. State clearly and briefly the Fundamental Assumption of Statistical Mechanics.

2. State clearly and briefly the Second Law of Thermodynamics

3. State clearly and briefly the Equipartition Theorem. When is it usually valid? Give an example of when it is valid and another one of when it is not valid.

4. Consider a gas containing a 50-50 mixture of He atoms and O\(_2\) molecules. There are N particles total; half the particles are He atoms and half are O\(_2\) molecules. The gas is in equilibrium at a temperature T near room temperature.
   (a) Consider the average translational kinetic energy of a He atom (\(\overline{KE}_{\text{He}}\)) and the average translational kinetic energy of an O\(_2\) molecule (\(\overline{KE}_{\text{O}_2}\)) What is the ratio \(\frac{\overline{KE}_{\text{O}_2}}{\overline{KE}_{\text{He}}}\)?
   Give a numerical answer and be sure to show your reasoning. (No credit for writing an answer, without any justification.)

   (b) Consider the rms speed of an He atom (\(v_{\text{rms,He}}\)) and the rms speed of an O\(_2\) molecule (\(v_{\text{rms,O}_2}\)). What is the ratio \(\frac{v_{\text{rms,O}_2}}{v_{\text{rms,He}}}\)?

   (c) Derive a formula for the constant-volume heat capacity of this N-particle gas, at temperatures near room temperature. (Heat capacity \(C = \frac{Q}{\Delta T}\), not specific heat.)
5. An ideal monatomic gas undergoes the process shown in the pV diagram below. Assume that the only type of work done on the gas is quasistatic compression or expansion.

For one complete cycle, compute the following quantities. Express all results in terms of $p_1$, $p_2$, $V_1$, $V_2$. Be sure to indicate clearly the sign of your answer.

(a) the change in internal energy $\Delta U$ of the gas

(b) the net work $W$ done on the gas

(c) the net heat $Q$ added to the gas

(d) the entropy change $\Delta S$ of the gas

6. Counting Problem.
(a) 3 indistinguishable balls are placed in 6 distinguishable boxes, with a maximum of one ball in a box allowed. How many possible configurations are there?

(b) 3 indistinguishable balls are placed in 6 distinguishable boxes. Any number of balls are allowed in each box. How many possible configurations are there?
7. Recall that the latent heat of melting is \( L = 330 \text{ J/g} \) for water, and the specific heat capacity of liquid water is \( c = 1 \text{ cal/(g} \cdot \text{K)} = 4.2 \text{ J/(g} \cdot \text{K)} \). Suppose an ice cube of mass \( m = 20 \text{ g} \), originally at temperature \( 0 \, ^\circ\text{C} \) is left sitting on a kitchen table, where it gradually melts. The temperature of the kitchen is \( 20 \, ^\circ\text{C} \).

(a) Calculate the change in entropy of the water as the solid ice cube melts into liquid at \( 0 \, ^\circ\text{C} \).

(b) Calculate the change in entropy of liquid water (from the melted ice) as its temperature rises from \( 0 \, ^\circ\text{C} \) to \( 20 \, ^\circ\text{C} \).

(c) Calculate the change in entropy of the kitchen as it gives up heat to the melting ice/warming liquid water.

(d) Calculate the net change in entropy of the universe during this process. Is the net change positive, negative, or zero. Is this what you would expect?

8. Consider a quantum system with 3 non-degenerate levels spaced a distance \( \varepsilon \) apart as shown. (An example of such a system is a spin-one particle in a magnetic field which would have spin orientations \(+1, -0, \text{ and } -1\) and energies \( -\mu \text{B}, 0, \text{ and } +\mu \text{B}.\) Note that we place the zero of energy at the middle energy level (not at the ground state level which is the common convention). The system is equilibrium at temperature \( T \).

\[ \begin{align*}
\text{E} & \uparrow \\
+\varepsilon & \rule{2cm}{.5pt} \\
0 & \rule{2cm}{.5pt} \\
-\varepsilon & \rule{2cm}{.5pt}
\end{align*} \]

(a) What is the partition function for this system?

(b) Derive an expression for the average energy of the system.

(c) Make a qualitative sketch of the graph of average energy vs temperature for this system. (No calculations necessary and I don’t expect you to use the answer in part (b) to answer this part; just think about it and sketch what you expect the curve to look like.)

(d) Using your answer to part (b), derive an approximate expression for the average energy in the high temperature limit \( kT >> \varepsilon \). Is your answer consistent with your graph in part (c)?
9. Consider a degenerate gas of electrons in a 2-dimensional box of area $A = L^2$. (A 2-dimensional electron gas can actually be physically constructed in a device called a 2D quantum well. The electrons are free to move on the interface between two different materials, but cannot move away from the interface.) For a particle in a 2D box, the allowed energies are given by (the usual particle-in-a-box formula): $E = \frac{\hbar^2}{8mL^2} (n_x^2 + n_y^2)$. $N$ non-interacting electrons are placed in the box at $T=0$. Derive a formula for the Fermi energy of this system.

10. (a) Starting with the thermodynamic identity ($dU = TdS - etc.$) and the definition of the Gibb’s free energy, derive the thermodynamic identity for the Gibb’s free energy ($dG = ...$) and use this to show what derivative relates entropy $S$ to Gibb’s free energy $G$.

(b) Make a qualitative sketch of $G$ vs. temperature $T$ for water in the neighborhood of the boiling point (liquid to gas transition) at constant pressure $p = 1$ atm.

(c) Use the following data to estimate the temperature of the boiling point of water at $p = 1$ atm:

<table>
<thead>
<tr>
<th></th>
<th>$G_{\text{liq}}$</th>
<th>$G_{\text{gas}}$</th>
<th>$S_{\text{liq}}$</th>
<th>$S_{\text{gas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-238 kJ</td>
<td>-229 kJ</td>
<td>70 J/K</td>
<td>189 J/K</td>
</tr>
</tbody>
</table>

If your estimate does not agree with what you expect, explain why.

11. The ground state energy of the hydrogen atom is $E_0 = -13.6$ eV and it is doubly degenerate. The first excited energy level is $E_1 = -3.4$ eV and that level is 8-fold degenerate. In a gas of hydrogen atoms, at what temperature are these two energy levels equally populated?
12. Graphs of entropy vs. energy are shown for two separated objects, A and B. Both graphs are on the same scale. The energies of these two objects initially have the same value as indicated on the graphs. The objects are then brought into thermal contact and reach equilibrium. Explain what happens to the energies of the two systems and indicate on the graphs where the equilibrium energies are. Although this is a qualitative problem, please be as precise as possible in your answer.