Combining complex impedances

**Series:**

\[ Z_1, Z_2, \ldots, Z_n \]

\[ Z_S = \sum_{k=1}^{n} Z_k \]

Works for any combination of complex impedances.

**Example 1:**

\[ Z_1 = R_1, \quad Z_2 = R_2 \]

\[ \Rightarrow Z_S = R_1 + R_2 \]

**Example 2:**

\[ Z_1 = \frac{1}{i\omega C_1}, \quad Z_2 = \frac{1}{i\omega C_2} \]

\[ \Rightarrow \frac{1}{i\omega C_S}, \text{ with } \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \]

This is equivalent to

\[ Z_S = \frac{1}{i\omega C_S} = \frac{1}{i\omega C_1} + \frac{1}{i\omega C_2} \]
Parallel impedances:

\[ Z_1 = R_1, \quad R_2 = Z_2 \quad \Rightarrow \quad \frac{1}{Z_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \checkmark \]

This works for any parallel arrangement of complex impedances.

Example 1:

\[ Z_1 = R_1, \quad R_2 = Z_2 \quad \Rightarrow \quad \frac{1}{Z_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \checkmark \]

Example 2:

\[ C_1 \quad \text{and} \quad C_2 \quad \Rightarrow \quad Z_{1/2} = \frac{1}{i\omega C_{1/2}} \]

We know that this is equivalent to \( \frac{1}{1} C_p = C_1 + C_2 \)

\[ i\omega C_p = i\omega C_1 + i\omega C_2 \quad \checkmark \]

Example 3:

\[ R \quad \text{and} \quad C \quad \Rightarrow \quad \frac{1}{Z} = \frac{1}{R} + i\omega C \quad \checkmark \]
Revisit voltage dividers:

\[ V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2} \quad \text{(from last class)} \]

Same works for complex impedances.

\[ V_{\text{out}} = V_{\text{in}} \frac{Z_2}{Z_1 + Z_2} \quad \text{for any elements with } Z_1 \text{ and } Z_2. \]

Example:

\[ V_{\text{out}}(t) = V_{\text{in}}(t) \frac{R}{\frac{1}{iwC} + R} \]

The transfer function for this circuit is:

\[ H(w) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R}{\frac{1}{iwC} + R} \]

\[ H(w) \quad \text{"Unity gain"} \]
The Thévenin Theorem:

Any network of sources and impedances can be replaced by a single source and a single impedance.

![Diagram of complicated network to simple circuit](image)

1) $V_{th}(t)$ is the open circuit voltage (i.e. the voltage across $A$ and $B$ with no load connected between $A$ and $B$).

2) To find $Z_{th}$ replace all voltage sources by wires and remove all current sources (open circuits). The resulting impedance between $A$ and $B$ is $Z_{th}$.

The Norton equivalent: (similar to the above)

Any network of sources and impedances can be replaced by a single current source and a single impedance.

![Diagram of Norton equivalent](image)

1) To find $I_{N}$, connect a wire between $A$ and $B$. The current that flows through that wire is $I_{N}$.

2) Short all voltage sources and remove all current sources.

$Z_{AB} = R_{N} = R_{th}$ (same as above)
Example 1: A function generator

Box with a fully complicated circuit in it. "A" says "50 Ω"
BNC output

Note: When you connect a 50 Ω load, the voltage between A and B drops to $V_{th} \frac{2}{2}$.

Example 2: The Wheatstone Bridge:

The Wheatstone bridge is used to compare an unknown resistance with a known resistance. For this, the bridge is balanced such that $V_A = V_B$, i.e., $V_{th} = 0$.

$V_{th}$ is the open-circuit voltage between A and B.

$$V_{th} = V \cdot \frac{R_2}{R_1 + R_2} - V \cdot \frac{R_x}{R_s + R_x}$$

The bridge is balanced when $V_{th} = 0$:

$$\frac{R_2}{R_1 + R_2} = \frac{R_x}{R_s + R_x}$$

(Lab #2)