Physics 3330  Lecture 61  16 Sept 2003

Questions?

Extra lecture this Jan
off the Chaise

Attendance during lab "official" time
is mandatory. If there is a persisting
problem, attendance will be taken

- OP AMPS

   IDEAL AMPLIFIER →

   1) Doesn't Distort input

   2) Doesn't Distort output

   Quite hard for all possible frequencies
   But easy for a restricted range.

We will eventually build amplifiers
directly from transistors, but as these
can be touchy, we will use a more
reliable and easy-to-use product
called an "OP-AMP" (operational amplifier)
to learn about simple amp concepts
such as negative and positive feedback.

As before, we'll consider "ideal"
OP-AMP, then develop realistic model.
 Symbol \( v_+ + v_\text{offset} \) often omitted

\( v_+ \rightarrow v_0 \)

\( v_- \rightarrow v_0 \)

\( v_0 = A(v_+ - v_-) \)

\( A = "\text{open loop gain}" \) (not in a feedback loop)

\( A \approx 10^5 \)

Note components are rarely exact and often temperature dependent. In specs \( \Rightarrow \) circuit must take this into account.

\( v_+ \) and \( v_- \) inputs are open

\( \Rightarrow \) no current flows into opamp inputs (\( R = 10^6 \Omega \))

\( \Rightarrow \) large gain works over small input range
Now for our 1st Amp Circuit

Try

\[ V_{out} = A \left( V_{in} - 0 \right) = A V_{in} \]

\[ A \text{ is } \approx 10^5 \]

Actual Output

\[ -V_{supply} < V_{out} < +V_{supply} \]

\[ \approx 1^{st} \text{ Reality Check} \]

Frequency Response

Not Good!
Very narrow band range

G looks like RC low pass filter

\[ \approx 25 \text{ Hz} \]
We can model this behavior:

\[ A = \frac{A_0}{1 + i \frac{f}{f_0}} = \frac{f_0 A_0}{f_0 + i f} \]

\[ f_T = f_0 A_0 = \text{unity gain frequency} \]

Because \[ A = 1 = \frac{f_0 A_0}{f_0 + i f_T} \]

\[ |A| = \frac{f_0 A_0}{f_0 + 2f_0 A_0} = \frac{A_0}{1 + i A_0} \]

\[ A_0 \gg 1 \Rightarrow A = -i \quad |A| = 1 \]

Summary: Simple op amp by itself has

Good input impedance (\(10^{12}\)Ω)

Ok output impedance (\(\approx 40\)Ω)

Too much gain except for MV signals

Terrible frequency response

\( \Rightarrow \) What is typical freq range [20 - 20k audio] \(\rightarrow 20\mu Hz\) radio

Simple solution is to use negative feedback loop.

\( V_{in} \) changes \( V_{+} \), \( V_{+} \) changes \( V_{out} \)

\( V_{out} \) changes to decrease \((V_{+} - V_{-})\) (diff)
First take $R = \infty$, $R_F = 0$

Imagine $V_-$ input at some value $V_+ = V_-$

$V_{in}$ increases, $V_+$ increases so

$V_+ - V_- \text{ increases}$

$V_{out} = A(V_+ - V_-) \text{ increases until}$

$V_- = V_+ \text{ again } \neq 0$

But now $V_{out} \to 0$, unstable

So actually we want

$V_{out} = A(V_+ - V_-) \Rightarrow \text{just compensate for } V_+ - V_-$

$V_- = V_{out} \Rightarrow V_{out} = A(V_+ - V_{out})$

We want

$V_{out} = \frac{A}{1 + A} V_+ \Rightarrow V_+ = V_{in}$

$\Rightarrow$ No gain! Called a follower (useful!)

Now add $R_F, R$

$V_{out} = A(V_+ - V_-)$

$V_- = V_{out} \frac{R}{R + R_F}$

$V_+ = V_{in}$
\[ V_{out} = A \left( \frac{V_{in} - \frac{R}{R+R_F}}{R+R_F} \right) \]

\[ V_{out} \left( 1 + \frac{AR}{R+R_F} \right) = A \cdot V_{in} \]

\[ \frac{V_{out}}{V_{in}} = \frac{A}{1 + \left( \frac{R}{R+R_F} \right)A} = \frac{1}{A + \frac{R}{R+R_F}} \]

For \( A >> 1 \)

\[ \frac{V_{out}}{V_{in}} \approx \frac{1}{B + \frac{R}{R+R_F}} = \frac{R+R_F}{R+R_F + R} \]

Closed Loop Gain

\[ G = 1 + \frac{R_F}{R} \]

Non-Inverting Gain, because \( V_{out} \) same sign as \( V_{in} \)

Closed Loop Gain = 1 + \( \frac{R_F}{R} \)

Closed Loop Gain, independent of \( A \) (as long as \( A \) is large!)

> What about frequency response?

Go back to

\[ \frac{V_{out}}{V_{in}} = \frac{A}{1 + BA} \]

Then

\[ A \rightarrow \frac{A_0}{1 + \frac{i \cdot f}{f_0}} \]

so

\[ \frac{V_{out}}{V_{in}} = \frac{A_0/(1 + i \cdot f/f_0)}{1 + \frac{BA_0}{1 + i \cdot f/f_0}} \]
\[
\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + i \frac{f}{f_0} + A_0 B} = \frac{A_0}{1 + A_0 B + i \frac{f}{f_0}}
\]

\[
= \left( \frac{A_0}{1 + A_0 B} \right) \left( \frac{1 + i \frac{f}{f_0}}{1 + i \frac{f}{f_0} (1 + A_0 B)} \right)
\]

Let \( G_0 = \frac{A_0}{1 + A_0 B} \) (low freq gain)

\[
f_B = \text{band width} = f_0 (1 + A_0 B)
\]

\[
G = \frac{V_{out}}{V_{in}} = \frac{G_0}{1 + i \frac{f}{f_0}}
\]

\[
\begin{align*}
G_0 & \quad \text{dB} \left(f \approx f_0 + 3 \text{dB} \right) \gg G = 0.70 + G_0 \\
\end{align*}
\]

\[f_B\]

\[
\text{Note:} \quad f_B \gg f_0 \quad (1 + A_0 B) \gg 1
\]

\[
\Rightarrow \quad G_0 \approx 1 + \frac{R_c}{R} = \frac{1}{B} \quad \text{"trading" bandwidth for gain}
\]

\[B \text{ large} \Rightarrow f_B \Rightarrow \text{large} \quad \text{G smaller}
\]
Don't forget Freq Response - sine waves
Stop function \( \Rightarrow \exp \)

\[ v_{in} \sqrt{\frac{\text{vout}}{1 - e^{-\frac{t}{\tau}}}} \]

\[ \tau = \frac{1}{2\pi f_B} \]

Note also \( C_0 = \frac{A_v}{(1 + A_v B) f_0} = \frac{f_T}{f_B} \)

\[ C_0 f_B = f_T \]

\[ f_T \approx 5 \text{MHz} \]

\[ C_0 = \frac{f_T}{f_B} \quad \text{or} \quad f_B = \frac{f_T}{C_0} \]

\[ f_B = f_T \]

\[ C_0 = 100 \Rightarrow f_B = \frac{5 \text{MHz}}{100} = 50 \text{kHz} \]

\[ B = 1, \quad C_0 = \frac{A_v}{1 + A_v} = 1 \]

\[ f_B = f_T \]
Input to output for non-inverting and inverting model

\[ V_{in} \rightarrow V_+ \]
\[ V_+ \rightarrow \frac{1}{2} R_i \]
\[ R_F \]
\[ V_{out} \]

At input
\[ i_m = \frac{V_+ - V_-}{R_i} = \frac{V_{in} - \frac{B V_{out}}{R_i}}{R_i} \]

\[ V_{out} = \frac{\left( \frac{A}{1 + AB} \right) V_{in}}{1 + AB} \]

\[ i_m = \frac{V_{in} - \frac{AB}{1 + AB} V_{in}}{R_i} \]

\[ V_{no} \rightarrow \frac{1}{R_i^\prime} = \frac{\left( \frac{1}{1 + AB} - \frac{AB}{1 + AB} \right) V_{in}}{1 + AB R_i} \]

\[ = \frac{V_{ir}}{(1 + AB) R_i} \]

\[ (i = \frac{v}{R}) \]

\[ R_i^\prime = (1 + AB) R_i \]
For output, use trick

\[ i_o = V - \frac{A(n_+ - n_-)}{R_0} \]

\[ n_+ = 0 \]
\[ n_- = V_B \]

\[ i_o = V - \frac{-AVB}{R_0} = \frac{V(1 + AB)}{R_0} \]

So

\[ R_0' = \frac{R_0}{1 + AB} \]

\[ R_0 \approx 40R \quad R_0' \ll 1R \]

What does this mean?

N-I-Amp looks like

\[
\begin{bmatrix}
\frac{R_0}{1 + AB} \\
N_{out}
\end{bmatrix}
\]