Physics 3320  Wed. Nov. 7, 2001  J. Ye

Outline: The Geometry of relativity

- Relativity of simultaneity
- Relativity of time interval — Time dilation
- Relativity of length scale — Lorentz contraction

Everything has become relative, the time, the distance, ...
The only thing remains the same is called "event interval" which we will see later as a scalar product of 4-vectors (space-time) in the structure of spacetime — a direct result of causality.

1. Relativity of simultaneity

Two events that are simultaneous in one inertial system are not, in general, simultaneous in another.

A simple interpretation:

If two events occur simultaneously at different locations in one inertial frame, then it means the two events can not have causality in between. Therefore the concept of simultaneity must be relative.

(nothing is violated if event A occurs earlier than B in another inertial frame, they are, after all, not related.)

Example:

\[ \begin{array}{c}
A \quad \quad \quad \quad \quad \quad \quad \quad B \\
\text{S frame} \\
\end{array} \]

The two clocks that are located equal-distance from the light source receive the light signal simultaneously in the frame S.

Now suppose we're in frame S that moves w.r.t. frame S to the right.
The light signal will strike clock B first before A.
**Time Dilation**

Reference frame $S$ is with the cart, moving at a speed $v$ with respect to the reference frame $S'$.

Calculate the time it takes for a light ray that leaves the bulb (distance $h$ above the floor of the cart) to reach the floor.

In frame $S$, $\Delta t = \frac{h}{c}$

In frame $S'$, total distance the light traveled is $\sqrt{h^2 + (Vt)^2}$

$$\Delta t' = \frac{\sqrt{h^2 + (Vt)^2}}{c}$$

$$(\Delta t')^2 = \frac{h^2 + (Vt)^2}{c^2}$$

$$\Delta t' = \frac{h^2}{c^2 - v^2} \cdot \frac{1}{1 - \frac{v^2}{c^2}} \cdot \Delta t$$

$$= \Delta t \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

With $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$

we have $\Delta t = \Delta t' \cdot \gamma$

Therefore the moving clock (time interval $\Delta t'$) runs slower than the stationary clock ($\Delta t$).
After learning about Lorentz transformation, we will come back on time dilation and prove that it is completely consistent with the principle of relativity.

- **Length (Lorentz) Contraction**

  Again, two reference frames \( S \) and \( \bar{S} \)

  \( S \) travels with the cart, \( \bar{S} \) refers to the ground

  ![Diagram](image)

  A lamp at the back of the cart, a mirror at the front

  In frame \( \bar{S} \),
  
  The round trip time for light to leave the lamp, hit the mirror, get reflected to the back is

  \[
  \bar{t} = \frac{2 \bar{x}}{c}, \quad \bar{x} : \text{length of the cart measured in frame } \bar{S}.
  \]

  For an observer in frame \( S \),

  ![Diagram](image)

  For light to go from the lamp to the mirror

  \[
  dt = \frac{\bar{x} + v \bar{t}_1}{c}
  \]

  where \( \bar{x} \) is the length of the cart measured in frame \( \bar{S} \).

  On the way back,
\[ \Delta t_1 = \frac{\Delta x - V \Delta t_1}{c} \]

Solve for \( \Delta t_1 \) and \( \Delta t_2 \), we have

\[ \Delta t_1 c = \Delta x + V \Delta t_1 \Rightarrow \Delta t_1 = \frac{\Delta x}{c-V} \]
\[ \Delta t_2 c = \Delta x - V \Delta t_2 \Rightarrow \Delta t_2 = \frac{\Delta x}{c+V} \]

So the total round-trip time is

\[ \Delta t = \Delta t_1 + \Delta t_2 = \frac{\Delta x}{c-V} + \frac{\Delta x}{c+V} \]
\[ = \Delta x \frac{c+V+c-V}{c^2-V^2} \]
\[ = 2 \Delta x \frac{c}{c^2-V^2} \]
\[ \therefore \Delta t = 2 \frac{\Delta x}{c} \frac{1}{1-V^2/c^2} \]

Combined with \( \Delta t = 2 \frac{\Delta x}{c} \)

we have \( \frac{\Delta t}{\Delta x} = \frac{AX}{\Delta x} \frac{1}{1-V^2/c^2} \)

but we already know \( \frac{\Delta t}{\Delta x} = \frac{1}{\sqrt{1-V^2/c^2}} \)

\[ \therefore \frac{\Delta x}{\Delta x} = \sqrt{1-V^2/c^2} \]

or \( \Delta x = \frac{1}{\sqrt{1-V^2/c^2}} \Delta x \)

\[ \Rightarrow \Delta x = \frac{\Delta x}{\gamma} \leq \Delta x \]

Length measurement for a moving object < length measurement for an object at rest
HW #8  Hints

[12.2]  
(a) \( m_A \vec{u}_A + m_B \vec{u}_B = m_C \vec{u}_C + m_D \vec{u}_D \)

Try to prove \( m_A \vec{u}_A' + m_B \vec{u}_B' = m_C \vec{u}_C' + m_D \vec{u}_D' \)

(we know that \( m_A + m_B = m_C + m_D \))

(b) From \( \frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 = \frac{1}{2} m_C u_C^2 + \frac{1}{2} m_D u_D^2 \)

Try to prove \( \frac{1}{2} m_A u_A'^2 + \frac{1}{2} m_B u_B'^2 = \frac{1}{2} m_C u_C'^2 + \frac{1}{2} m_D u_D'^2 \)

[12.3]  
(a) \((V_{AC})_{Galileo} = V_{AB} + V_{BC}\)

\((V_{AC})_{Einstein} = \frac{V_{AB} + V_{BC}}{1 + \frac{V_{AC} V_{BC}}{c^2}}\)

Calculate \(\frac{(V_{AC})_{Galileo} - (V_{AC})_{Einstein}}{(V_{AC})_{Galileo}}\)

(b) Use eq (12.3)

(c) Suppose you use \( \beta = \frac{V_{AC}}{c} \)

Try to prove \( 1 - \beta^2 > 0 \)

(You can save a lot of writing by using symbols such as \( \beta_1 = \frac{V_{AC}}{c}, \beta_2 = \frac{V_{BC}}{c} \))

[12.6]  

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\( \vec{v} \quad \vec{a} \quad \theta \\)  

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To Earth
Want the quantity $\Delta s/\Delta t$.

$\Delta s = v \Delta t \sin \theta$

We need $\Delta t$!

Suppose:
- Light signal leaves $a$ at time $t_a'$, arrives at Earth at time $t_a = t_a' + \Delta s/c$
- Light signal leaves $b$ at time $t_b'$, arrives at Earth at time $t_b = t_b' + \Delta b/c$

$\Delta t = t_b - t_a$
$\Delta t' = t_b' - t_a'$

$\Delta b - \Delta a = \Delta s = (v \Delta t') \cos \theta$

From these expressions you will find the relation between $\Delta t$ and $\Delta t'$.

[12.7] Due to time dilation, in the lab frame the muon is going to live longer than its proper lifetime in a rest frame.

The total distance traveled in the lab

\[ d = \text{muon velocity} \times \text{prolonged lifetime} \]

depends on $v$ depends on $v$

Then solve for $V$. 