Classical (Galilean) principle of relativity:

The basic laws of mechanics have the same form in any inertial reference frame, i.e., coordinate systems moving with constant velocity with respect to each other.

We have two reference systems $S$ and $S'$ with a relative velocity $\overrightarrow{V}$, such that $\overrightarrow{R_0} = \overrightarrow{V}t$.

$\overrightarrow{R}_0$ is the relative position of the two origins.

Observation point $P$:

it has a position vector $\overrightarrow{r}$ in frame $S$ and $\overrightarrow{r}'$ in frame $S'$.

Therefore $\overrightarrow{r}' = \frac{d\overrightarrow{r}}{dt} = \frac{d(\overrightarrow{r} + \overrightarrow{R}_0)}{dt} = \overrightarrow{v} - \overrightarrow{V}
so the velocities of the point $P$ as measured in the two systems are related by the familiar Galilean velocity addition rule.

Since $\overrightarrow{V}$ is a constant, $\frac{d\overrightarrow{V}}{dt} = 0$.

$\therefore \frac{d\overrightarrow{v}'}{dt} = \overrightarrow{a}' = \frac{d\overrightarrow{v}}{dt} = \overrightarrow{a}$.
Therefore, the force on a point mass $m$ located at $E$ are also equal,

$$F' = m\vec{a}' = m\vec{a} = \vec{F}.$$  

One mechanical phenomena, the same interpretation from $S$ and $S'$.

How about electromagnetism?

In the discussion of Faraday's law, we know the electric fields in two frames with relative velocity $\vec{V}$ are related by

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B}.$$  

This expression has an explicit dependence on $\vec{V}$. The same phenomena is described in different ways in two inertial frames.

As an example, $\vec{E} = 0$ in frame $S$

Then $\vec{E}' = \vec{V} \times \vec{B}$.

A pure magnetic field $\vec{B}$ in $S$ is seen as a pure electric field $\vec{E}'$ in $S'$.

Another example:

From Maxwell's equation, we have derived the wave equation

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0.$$  

Let's suppose the plane wave is propagating along $\hat{x}$ in frame $S$, and frame $S'$ is moving with $\vec{V} = V\hat{x}$

$$\therefore \begin{cases} x' = x - Vt \\ y' = y \\ z' = z \\ t' = t \end{cases} \quad \Rightarrow \quad \text{Galilean transformation}$$

We start from $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$ in frame $S$. 
Assume \( \psi = \psi(x', t') \)

\[
\frac{\partial \psi}{\partial x} = \frac{\partial x'}{\partial x'} \frac{\partial \psi}{\partial x} + \frac{\partial x'}{\partial t'} \frac{\partial \psi}{\partial t'}
\]

\[
= \frac{\partial x'}{\partial x'}
\]

and therefore \( \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi}{\partial x'^2} \)

Then \( \frac{\partial \psi}{\partial t} = \frac{\partial x'}{\partial t'} \frac{\partial \psi}{\partial t'} + \frac{\partial x'}{\partial t'} \frac{\partial \psi}{\partial t'} \)

\[
= -V \frac{\partial \psi}{\partial x'} + \frac{\partial \psi}{\partial t'}
\]

and \( \frac{\partial^2 \psi}{\partial t^2} = -V \frac{\partial}{\partial x'} \left( \frac{\partial \psi}{\partial t'} \right) + \frac{\partial}{\partial t'} \left( \frac{\partial \psi}{\partial t'} \right) \)

\[
= -V \frac{\partial}{\partial x'} \left(-V \frac{\partial \psi}{\partial t'} + \frac{\partial \psi}{\partial t'} \right) + \frac{\partial}{\partial t'} \left(-V \frac{\partial \psi}{\partial t'} + \frac{\partial \psi}{\partial t'} \right)
\]

\[
= V^2 \frac{\partial \psi}{\partial x'^2} - 2V \frac{\partial \psi}{\partial x' \partial t'} + \frac{\partial \psi}{\partial t'^2}
\]

Hence we have a new wave equation in reference frame \( s' \):

\[
\frac{\partial^2 \psi}{\partial x'^2} = \frac{1}{c^2} \left( V^2 \frac{\partial^2 \psi}{\partial x^2} - 2V \frac{\partial \psi}{\partial x \partial t'} + \frac{\partial \psi}{\partial t'^2} \right)
\]

or \( (c^2 - V^2) \frac{\partial^2 \psi}{\partial x'^2} = \frac{\partial^2 \psi}{\partial t'^2} - 2V \frac{\partial \psi}{\partial x' \partial t'} \).

We can still find a plane wave solution in frame \( s' \):

\[
\psi(x', t') = \psi_0 e^{i k' (x' - u't')} \]

with the wave velocity \( u' = \pm (c \mp V) \).

If the wave travels in the positive \( x' \) direction, \( u' = c - V \)

if the wave travels in the negative \( x' \) direction, \( u' = -(c + V) \).
These examples tell us that:

- If Galilean transformation and Maxwell's equations are both right, then electromagnetic effects will generally be different when observed from different inertial systems.

- The speed of a plane wave in vacuum would not always be \( c \).
  It is true for only one frame: Absolute rest frame.
  This implies that Maxwell's equations have the proper forms only in that rest frame.

- The absolute rest frame is the primary inertial system of mechanics, and supports wave propagation, \( \text{ETHER} \).
  All other frames should be able to detect motion of \( \text{ETHER} \).
  \( \text{ETHER} \) wind.

- Principle of relativity is so simple and beautiful for mechanics, maybe overall there exists a common principle of relativity valid for both mechanics and electromagnetism.
  The Galilean principle of relativity and Newtonian Mechanics are no longer appropriate.

  Let experimental results decide!
For the arm that is parallel to \( \vec{V} \) total time for a round trip through the arm is

\[
t_1 = \frac{1}{c-V} + \frac{1}{c+V} \approx \frac{2l}{c} \left(1 + \frac{V^2}{c^2}\right)
\]

For the perpendicular arm, \( \vec{V} \parallel \vec{C} \)

\[
v' = \sqrt{c^2 - V^2}
\]

\[
t_2 = \frac{2l}{\sqrt{c^2 - V^2}} \approx \frac{2l}{c} \left(1 + \frac{V^2}{2c^2}\right)
\]

\[
\Delta t = t_1 - t_2 \approx \frac{l}{c} \left(\frac{V}{c}\right)^2
\]

This time difference was NEVER observed, with experiments being pushed to higher and higher precisions.

So no absolute rest frame after all.

Einstein’s postulates:
1. The principle of relativity:
   - Laws of physics apply in all inertial frames
2. The universal speed of light:
   - \( c \) is the same for all inertial observers, regardless of the motion of the source.

These two postulates constitute of Einstein’s special relativity: we now have to use a new way to look at time, length, velocity, mass, energy, ... a profound change in physics.

For example, \( V' = V - V \)

Galileo’s velocity addition

\[
\text{now changes to} \quad v' = \frac{v-V}{1 - vv/c^2} \quad \text{Einstein’s velocity addition}
\]