Homework 5  
(Due Date: 5 pm Wed. Feb 11 2015, at PHYS2210 slots on the Wooden box in Helproom )

NOTE: Show your work neatly, explain what you are doing. Correct answers, for which we cannot follow the work, will get ZERO credit!

1. In Taylor’s book, he simplifies Eqn 2.42 for the range of a projectile by assuming that $R^2 \approx R_{vac}^2$, which is a bit unsatisfying. Show that if you use the quadratic equation to solve for $R$ in Eqn 2.42, and make the appropriate Taylor series expansion for the square root part (keeping terms up to $v_0^2$) that you obtain the expression given in Equation 2.44.

2. An object is released from rest far above the ground. The object experiences a force due to the air resistance of which magnitude is proportional to the objects speed $\vec{F} = -bv$. In this problem let’s define the +y-axis to point down.

   (a) Your friend, who is also taking Classical Mechanics, has calculated $v_y(t)$. He knows that the terminal velocity of the object is 4 m/s, and has found a solution, $v_y(t) = 4(1 - e^{-t/2})$, but isn’t confident that it is right. Check the values of the proposed formula at $t = 0$ and $t \to \infty$. Does his/her equation give the expected values so far?

   (b) Given that the drag force on this object is $\vec{F} = -bv$, the drag force is not very significant for small times. What is the formula for $v_y(t)$ in the case of no drag?

   (c) Given the above, check your friends solution by finding an approximate form that is valid for small t. If you find that it is incorrect, can you suggest to your friend what term(s) in the equation he/she might want to double check?

3. A car at an initial speed $v_0$, begin to brake hard on a flat road at $t = 0$. There are two significant resistive forces acting on it, a quadratic ($cv^2$) air drag, and a constant frictional force ($\mu mg$). If you are interested in finding $v(x)$, rather than $v(t)$, there is a commonly used method known as the “$v \ dv/dx$ rule”, which uses the chain rule to rewrite $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$.

   (a) Write down the equation of motion for $\dot{v} = f(v)$ and use the “$v \ dv/dx$” rule to solve the equation of motion directly for $v(x)$, and show that the distance the car needs for a full stop is:

   $$x_{max} = \frac{A^2}{2\mu g} \ln \left( \frac{A^2 + v_0^2}{A^2} \right)$$

   here $\mu$ is the friction coefficient. What is the constant $A$ in this case (in terms of given parameters in the differential equation?)

   (b) My car’s mass is estimated $m \approx 1600$ kg, and I estimate $c \approx 0.6$ kg/m. I’m driving down I-70 towards Glenwood Springs, traveling 75 mph, when I see a stopped car on the highway 300 m front of me. I slam on the brakes! Assuming a friction coefficient of $\mu = 0.7$, find my stopping distance (do I crash, or am I safe?), and then compare it to what it would have been in the absence of any air drag (i.e. just the road friction alone). Briefly, comment. If I’d been driving a futuristic sports car shaped like Boeing747, traveling at twice this speed (but with all other parameters the same) would air drag have made a more significant difference? Explain your answer.

   (c) Car engines are complicated, they are not simple “constant force” devices, but I estimate that at highway speeds my car might apply a fairly steady forward force against the road of about 1500 N. If the stopped car managed to pull off to the side of the highway when I had slowed to 30 mph and I immediately started re-accelerating with this constant maximum forward force, how much highway distance would I need to get my car back up to 75 mph? (Include the quadratic air resistance, of course!) (Hint: the $v \ dv/dx$ rule may be useful again.)

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(d) Make a rough sketch (by hand, not with Mathematica!) of my car’s velocity \( v(t) \), i.e. velocity as a function of time, described by the “story” of parts (b) and (c). Note: that is \( v(t) \), not \( v(x) \). Don’t calculate it, just sketch what it should look like. Comment on interesting features of your graph (e.g., signs of slope, signs of concavity, interesting points...)

4. Taylor series are the single most important and common approximation technique used throughout physics. We need to keep practicing with them.

(a) Suppose \( f(x) = 1/(1 + x) \). Find an approximate expression for \( f(x) \) by doing the expansion near \( x = 0 \) (i.e. Maclaurin Series), going out to second order. Use it to estimate \( f(x = 0.1) \) and \( f(x = 5) \). In both cases, compare the exact result for \( f(x) \) with your series approximation, going to first order (terms proportional to \( x \)) and also second order (\( x^2 \)). Does your estimate improve at second order in both cases? Comment on the general criterion about \( x \) you would guess tells you when the series approach seems to be a fruitful one here.

(b) For a given function \( f(x) = (1-x)/(1+x) \), do the Taylor-expansion to approximate \( f(x) \) out to second order. There are many ways to do this. First, use the usual formal formula for Taylor expansion. Another way is to take your series answer in part a, and multiply it by \( (1-x) \). Try both ways and use it to check yourself, both methods should agree to all orders! Which method do you prefer?

(c) Now let \( f(x) = (a-x)/(a+x) \), where \( a \) is a constant with units, like \( a = 5 \) m. Expand to second order. In this case, do NOT start “from scratch” blindly using the math formula. Instead, factor out “\( a \)”, so that it looks exactly like what you had in part b, except the “thing you’re expanding in” isn’t \( x \). What is it? For this part, what is now the general criterion on \( x \) that tells you when this series approach seems to be a fruitful one? Why? (And please do not say “\( x \) must be small” or “\( x \ll 1 \). Small compared to what? You cannot compare meters to numbers, or apples to oranges!)

5. An astronaut has drifted too far away from the space shuttle while attempting to repair the Hubble Space telescope. She realizes that the orbiter is moving away from her at 3 m/s. She and her space suit have a mass of 90 kg. On her back is a 10 kg jetpack which consists of an 8 kg holding tank filled with 2 kg of pressurized gas. She is able to use the gas to propel herself directly towards the orbiter. The gas exits the tank at a uniform rate with a constant velocity of 100 m/s, relative to the tank (and her).

(a) After the tank has been emptied, what is her velocity? Will she be able to catch up with the orbiter with that velocity?

(b) With what velocity (in her frame of reference!) will she have to throw the empty tank away to reach the orbiter?

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