Today's class:

- Superposition States
- 'Measurements'

Energy Eigenstates

- Solve Schrödinger equation:
  \[-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x) \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}\]
- Get solutions for a bunch of different energies: \(E_1, E_2, E_3, \ldots\) (called 'Energy Eigenstates.' \(E_n\): 'Eigenvalues')
- Different 'eigenstates' for each energy:
  \(\psi_1(x), \psi_2(x), \psi_3(x), \ldots\)
  \(\Psi_1(x,t) = \psi_1(x)e^{-iE_1t/\hbar}, \Psi_2(x,t) = \psi_2(x)e^{-iE_2t/\hbar}, \ldots\)
- State with a single energy is called an "energy eigenstate."

Examples of Energy Eigenstates

- Free Particle
  \(\psi(x) = e^{ikx} \text{ or } e^{-ikx}\)
  \(E = (\hbar k)^2/(2m)\)
- Infinite Square Well/Rigid Box
  \(\psi_n(x) = \frac{1}{\sqrt{L}} \sin(n\pi x/L)\)

Reminder… Superposition

Review of Time Dependence

Say an electron were in the state \(\Psi(x,t) = \Psi_i(x,t)\)
where \(\Psi_i(x,t)\) is the wave function for the ground state of the infinite square well.

Remember: You can always write an energy eigenstate as
\(\Psi(x,t) = \psi(x)e^{-iEt/\hbar}\).

Probability density = \(|\Psi(x,t)|^2 = \Psi(x,t)\Psi^*(x,t)\)
\(= \psi(x)e^{-iEt}\psi^*(x)e^{iEt/\hbar} = \psi(x)\psi^*(x) = |\psi(x)|^2\)
\(\Rightarrow\) wave function has time dependence in phase.
\(\Rightarrow\) probability density has no time dependence.

Time dependence of wave function is not observable. Only probability density is observable.
Time Dependence of Superposition States

An electron is in the state
\[ \Psi(x,t) = \frac{1}{\sqrt{\beta}} \Psi_1(x,t) + \frac{1}{\sqrt{\beta}} \Psi_2(x,t), \]
where
\[ \Psi_1(x,t) = \psi_1(x)e^{-iE_1t/\hbar} \]
and
\[ \Psi_2(x,t) = \psi_2(x)e^{-iE_2t/\hbar} \]
are the ground state and first excited state of the infinite square well. (Note: \( \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1 \))

Does the probability density of the electron change in time?

Measurement

- Measurement is a discontinuous process, not described by the Schrödinger equation. (Schrödinger describes everything before and after, but not moment of measurement.)
- If you measure energy of particle, will find it in a state of definite energy (= energy eigenstate).
- If you measure position of particle, will find it in a state of definite position (= position eigenstate).
- Unlike classical physics, measurement in QM doesn’t just find something that was already there – it CHANGES the system!

Note on energy

- Measuring energy ... why \( P(E_n) = |c_n|^2 \)
- \( \Psi(x,t) = c_1 \Psi_1(x,t) + c_2 \Psi_2(x,t) + c_3 \Psi_3(x,t) + \ldots \)
- \( \Psi_n(x,t) = c_n \Psi_n(x,t) \)
- \( \int_{-\infty}^{\infty} \Psi_n^*(x,t) \Psi_n(x,t) dx = \int [c_n^* \Psi_n^*(x,t) \Psi_n(x,t)] dx \)

But sines are orthogonal functions that is:
- unless \( n = m \)
- \( = 1 \) if \( n = m \)
- \[ \int_{-\infty}^{\infty} \Psi_n^*(x,t) \Psi_n(x,t) dx = 1 \]
- \[ \int_{-\infty}^{\infty} c_n \Psi_n^*(x) \Psi_n(x) dx = \int_{-\infty}^{\infty} c_n \Psi_n^*(x)c_n \Psi_n(x) dx = |c_n|^2 \]

How to compute the probability of measuring a particular state:

Suppose you have a particle with wave function
\[ \Psi(x,t) = c_1 \Psi_1(x,t) + c_2 \Psi_2(x,t) + c_3 \Psi_3(x,t) + \ldots \]

- Measuring position:
  \[ P(a \text{ to } b) = \int_a^b |\Psi(x,t)|^2 dx \]
- Measuring energy:
  \[ P(E_n) = |c_n|^2 \]

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Von Neumann Postulate: If you make measurement of particle in a state \( \Psi(x,t) \), the probability of finding particle in a state \( \Psi_a(x,t) \) is given by:
\[ \int_{-\infty}^{\infty} \Psi_a^*(x,t) \Psi(x,t) dx = \text{overlap between } \Psi \text{ & } \Psi_a \]

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Measuring position

- Example: double slit experiment:
- Probability density at screen looks like:
- Probability of measuring particle at particular pixel:
  \[ P = \int_a^b |\Psi(x,t)|^2 dx \]
Note on position and energy measurements:

- Energy eigenstates tend to be spread out in space.
- Position eigenstates tend to be localized in space.
- This is why you can’t know both at the same time (wave packets vs. plane waves)
- Measuring position messes up energy eigenstate and vice versa.

That’s all on superposition states
...and all about the 1D Schrödinger equation.

Up next:
Schrödinger in 2D and 3D