Announcements:

Reading for Friday: 11.9 & 11.10

Exam 2 is next week (Thu. 7:30-9pm)

Topics: TZD, Chapters 3-5, 11.9, 11.10

HW07 due Fri. Oct 19

HW08 will be due Wed. Oct. 30

DR helproom Hrs: Thu 1-2 this week only

Summary of important Ideas

1) Electrons in atoms are found at specific energy levels
2) Different set of energy levels for different atoms
3) **One** photon emitted per electron jump down between energy levels. Photon color determined by energy difference between the two levels.
4) If electron not bound to an atom: Can have any energy. (For instance kinetic energy of free electrons in the PE effect.)

<table>
<thead>
<tr>
<th>Hydrogen</th>
<th>Lithium</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Energy Levels](not to scale)</td>
<td>![Energy Levels](not to scale)</td>
</tr>
<tr>
<td>Electron energy levels in 2 different atoms: Levels have different spacing (explains unique colors for each type of atom.</td>
<td>Atoms with more than one electron: lowest levels filled.</td>
</tr>
</tbody>
</table>

Quiz on the reading

(no talking and closed book)

The Rydberg formula describes

a. the colors of light emitted from all types of atoms.
b. the angular distribution of particles scattering off of atoms that Rutherford observed in his famous experiment.
c. the colors of light emitted by hydrogen atoms.
d. the energy distribution of cathode rays
e. the frequencies of light emitted by helium atoms.
Now we know about the energy levels in atoms. But how can we calculate/predict them?

→ Need a model

Step 1: Make precise, quantitative observations!
Step 2: Be creative & come up with a model.
Step 3: Put your model to the test.

Balmer series: A closer look at the spectrum of hydrogen

410.3 486.1 656.3 nm
434.0

Balmer (1885) noticed wavelengths followed a progression

\[ \lambda = \frac{91.19 \text{nm}}{\frac{1}{2^2} - \frac{1}{n^2}} \]

where \( n = 3, 4, 5, 6, \ldots \)

As \( n \) gets larger, what happens to wavelengths of emitted light?

→ \( \lambda \) gets smaller and smaller, but it approaches a limit.

Hydrogen atom – Rydberg formula

Does generalizing Balmer’s formula work?
Yes! → It correctly predicts additional lines in HYDROGEN.

Rydberg’s general formula

\[ \lambda = \frac{91.19 \text{nm}}{\frac{1}{2^2} - \frac{1}{n^2}} \]

Predicts \( \lambda \) of \( n \rightarrow m \) transition:

\( n \quad (n > m) \)
\( m \quad (m = 1, 2, 3, \ldots) \)

\( \lambda_{\text{limit}} = 4 \times 91.19 \text{nm} = 364.7 \text{nm} \)

\( \lambda \) gets smaller and smaller, but it approaches a limit.
Hydrogen atom – Lyman Series

Rydberg’s formula

\[ \lambda = \frac{91.19 \text{nm}}{m^2 - \frac{1}{n^2}} \]

Predicts \( \lambda \) of \( n \rightarrow m \) transition:

\( n \quad (n>m) \)

\( m \quad (m=1,2,3...) \)

Can Rydberg’s formula tell us what ground state energy is?

Looking for Ground State Energy

Balmer-Rydberg formula

\[ \lambda = \frac{91.19 \text{nm}}{m^2 - \frac{1}{n^2}} \]

Look at energy for a transition between \( n=\infty \) and \( m=1 \)

\[ E_{\text{initial}} - E_{\text{final}} = \frac{hc}{\lambda} = \frac{hc}{91.19 \text{nm}} \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \]

\[ -E_{\text{final}} = \frac{hc}{91.19 \text{nm}} \]

\[ E_1 = -13.6 \text{eV} \]

A more general case: What is the energy of each level (‘m’) in hydrogen?

\[ E_m = -13.6 \text{eV} \frac{1}{m^2} \]

Balmer/Rydberg had a mathematical formula to describe hydrogen spectrum, but no mechanism for why it worked!

Why does it work?

Hydrogen energy levels

\( \infty \rightarrow 0 \text{eV} \)

\( 6 \rightarrow 5 \)

\( 4 \rightarrow 3 \)

\( 3 \rightarrow 2 \)

\( 2 \rightarrow 1 \)

U\( V \)

\( E_1 = -13.6 \text{eV} \)

\( 410.3 \text{ nm} \) 486.1 \text{ nm} 656.3 \text{ nm} \)

\( \lambda = \frac{91.19 \text{nm}}{m^2 - \frac{1}{n^2}} \)

where \( m=1,2,3 \)

and where \( n = m+1, m+2 \)
The Balmer/Rydberg formula correctly describes the hydrogen spectrum!

Is it a good model?

The Balmer/Rydberg formula is a mathematical representation of an empirical observation.

It doesn’t really explain anything.

How can we explain (not only calculate) the energy levels in the hydrogen atom?

Next step: A semi-classical explanation of the atomic spectra (Bohr model)

Electrostatic potential energy

When an electron moves to location further away from the nucleus its energy increases because energy is required to separate positive and negative charges, and there is an increase in the electrostatic potential energy of the electron.

→ Force on electron is less, but Potential Energy is higher!

→ Electrons at higher energy levels are further from the nucleus!

Potential energy of the electron in hydrogen

We define electron’s PE as 0 when far, far away from the proton!

$\int F \cdot dr = \int_{\infty}^{D} \frac{kq_{e}q_{prot}}{r^2} dr$

$PE = kq_{e}q_{prot} \int_{\infty}^{D} \frac{dr}{r^2} = kq_{e}q_{prot} \frac{1}{r} \bigg|_{\infty}^{D} = -\frac{ke^2}{D}$

(k = $\frac{1}{4\pi\varepsilon_0}$; Coulomb force const.)
Potential energy of a single electron in an atom

PE of an electron at distance D from the proton is

\[ PE = -\frac{ke^2}{D}, \quad ke^2 = 1.440\text{eV} \cdot \text{nm} \]

Bohr Model

- When Bohr saw Balmer’s formula, he came up with a new model that would predict it and 'solve' the problem of electrons spiraling into the nucleus.
- The Bohr model has some problems, but it's still useful.
- Why doesn’t the electron fall into the nucleus?
  - According to classical physics, it should!
  - According to Bohr, it just doesn’t.
  - Modern QM will give a more satisfying answer, but you’ll have to wait till next week.


Bohr's approach:

#1: Treat the mechanics classical (electron spinning around a proton):

- Newton’s laws assumed to be valid
- Coulomb forces provide centripetal acceleration.

#2: Bohr’s hypothesis (Bohr had no proof for this; he just assumed it – leads to correct results!):

- The angular momentum of the electrons is quantized in multiples of \( \hbar \).
- The lowest angular momentum is \( \hbar \).

Bohr Model. #1: Classical mechanics

The centripetal acceleration

\( a = \frac{v^2}{r} \) is provided by the coulomb force \( F = k \cdot \frac{e^2}{r^2} \).

\( (k = \frac{1}{4\pi \varepsilon_0}) \): Coulomb force const.

Newton’s second law \( \Rightarrow mv^2/r = k \cdot \frac{e^2}{r^2} \)

\( \text{or} \Rightarrow \quad mv^2 = k \cdot \frac{e^2}{r} \)

The electron’s kinetic energy is \( KE = \frac{1}{2} \cdot m \cdot v^2 \)

The electron’s potential energy is \( PE = -ke^2/r = \frac{1}{2} PE \)

\( \Rightarrow E = KE + PE = -\frac{1}{2} ke^2/r = \frac{1}{2} PE \)

Therefore: If we know \( r \), we know \( E \) and \( v \), etc…
Bohr Model. #2: Quantized angular momentum

Bohr assumed that the angular momentum of the electron could only have the quantized values of:

\[ L = n\hbar \]

And therefore:

\[ mvr = n\hbar, \ (n=1,2,3,...) \]

Or:

\[ v = n\hbar/(mr) \]

Substituting this into \( mv^2 = k\cdot e^2/r \) leads to:

\[ r_n = r_Bn^2, \text{ with } r_B = \frac{\hbar^2}{ke^2m} = 52.9 \text{ pm} \]

Bohr radius

\[ E_n = E_R/n^2, \text{ with } E_R = \frac{m(ke^2)^2}{2\hbar^2} = 13.6 \text{ eV} \]

Rydberg Energy

Bohr Model. Results

\[ r = r_Bn^2, \text{ with } r_B = \frac{\hbar^2}{ke^2m} = 52.9 \text{ pm} \]

Bohr radius

\[ E_n = E_R/n^2, \text{ with } E_R = \frac{m(ke^2)^2}{2\hbar^2} = 13.6 \text{ eV} \]

Rydberg Energy

The Bohr model not only predicts a reasonable atomic radius \( r_B \), but it also predicts the energy levels in hydrogen to 4 digits accuracy!

Possible photon energies:

\[ E_i = E_n - E_m = E_R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \]

\[ (n > m) \]

→ The Bohr model 'explains' the Rydberg formula!!