Physics 2010
Written HW 6: Waterslide Wipeout – Conservation of Energy  (Due Mar 14)

In recent homeworks, you investigated the physics of the Megawoosh video. Here you will reinvestigate by applying the conservation of energy (with and without friction!) This will allow you to consider Adam’s slide down the larger ramp (Triangle 1) and up the smaller ramp (Triangle 2).
Data: \( \theta_1 = 24^\circ, L_1 = 50 \text{ m}, x_1 = 45.7 \text{ m}, h_1 = 20.3 \text{ m}, \)
For triangle 2: \( \theta_2 = 30^\circ, L_2 = 7.4 \text{ m}, x_2 = 6.4 \text{ m}, h_2 = 3.7 \text{ m}. \)

Adam starts from rest at the top of the larger triangle. Assume the kink between the triangles is smooth enough for him to make the turn gracefully. The coefficient of friction \( \mu_k \) of both ramps is the same.

1. Below, I started to draw a plot of Adam’s speed \( v \) as a function of time for his slide down Triangle 1, assuming no friction (\( \mu_k = 0 \)). I stopped sketching at the time when he reaches the bottom of triangle 1: please continue the plot as he goes back up triangle 2 until take-off into the air. (Stop plotting there!) Then, add to the same figure a second, dashed line plot to show his speed assuming the slides do have friction (\( \mu_k \neq 0 \)).

We are not looking for any calculations here, just a qualitatively reasonable sketch.

Briefly discuss your reasoning for your sketch.

Various things to think and write about: When \( \mu_k = 0 \), how does the acceleration on ramp #2 compare with that on ramp #1? (The angles are not the same!) How does the time up ramp #2 compare to time down ramp #1? With friction, acceleration along a given ramp is still going to be constant. But, how does it compare with the acceleration without friction? And how does the acceleration UP the second ramp compare with acceleration DOWN the first ramp, given that there is friction and the angles are different?)
2. In written HW #4, we made the following claim:

“In the absence of friction, Adam’s speed at take off will be the same as if he had slid only to the point along Triangle 1 that has the same height as Triangle 2”

Briefly, justify this claim, using the principle of conservation of energy.

3. Assume that the slide is frictionless ($\mu_k = 0$): If Adam starts (at rest) at the top of slide #1, and ends up taking off from the top of slide #2, use conservation of energy to decide which expression below gives Adam’s speed $v$ at take-off into the air from the smaller ramp.

(A) $\sqrt{2gh_1}$  (B) $\sqrt{2gh_2}$  (C) $\sqrt{g(h_1 - h_2)}$  (D) $\sqrt{2g(h_1 - h_2)}$  (E) $\sqrt{2g(h_2 - h_1)}$

Present your reasoning.

4. Compare Adam’s mechanical energy at take-off from the smaller ramp with and without friction. Mechanical energy at take-off will be….

(A) … smaller if the ramp has friction  (B) … larger if the ramp has friction
(C) … the same, whether or not the ramp has friction  (D) … impossible to tell from the information given.

Very briefly, explain your reasoning….
For the remainder of questions, assume that the slide does have kinetic friction ($\mu_k \neq 0$).

5. Which of the following equations gives $W_{\text{friction on Adam, triangle 1}}$ for JUST the downward trip (going down triangle 1), assuming that Adam has mass $m$? *(Hints: Work is the component of force in the direction of displacement times the displacement. You will need to find the force of friction. Work is negative when the force acts opposite to the direction of motion)*

   (A) $-\mu_k mg L_1 \sin \theta_1$  
   (B) $+\mu_k mg L_1 \sin \theta_1$  
   (C) $-\mu_k mg L_1 \cos \theta_1$  
   (D) $+\mu_k mg L_1 \cos \theta_1$  
   (E) $-\mu_k mg L_1$

**Present your reasoning.**

6. Now derive an equation for $W_{\text{friction on Adam, triangle 2}}$, the work done by friction during Adam’s slide up the smaller ramp (Triangle 2)? *(Hint: your answer should look roughly similar to one of the options in the previous question, but of course with triangle #2 parameters)*  
**Present your reasoning.**

7. Let $W_{\text{friction on Adam}}$ be the total work done by sliding friction on Adam, from the top of the larger ramp to the top of the smaller ramp. (It will be the sum of the answers to the last two questions)  
Which of the following equations accurately states the conservation of energy for Adam’s motion from the start to the end, the top of the smaller ramp? ($v$ is his speed at the top of the smaller ramp)

   (A) $mgh_1 = \frac{1}{2}mv^2 + mgh_2 + W_{\text{by friction on Adam}}$  
   (B) $mgh_1 = \frac{1}{2}mv^2 + mgh_2 - W_{\text{by friction on Adam}}$  
   (C) Neither of the above two equations gets it right!

**Briefly, present your reasoning.** *(If your answer is C, what should it be and why?)*
8. Pull it all together to show that Adam’s speed at take-off, with friction ($\mu_k \neq 0$) will be:

$$v = \sqrt{2g(h_1 - h_2)} - 2g\mu_k(L_1 \cos \theta_1 + L_2 \cos \theta_2)$$

Present your reasoning.

9. If you watched the Mythbusters videos you saw that on Adam’s first trip down the water slide he nearly stopped even before reaching the top of the shorter ramp! Using the equation above (and using the numerical dimensions and geometry of the two ramps given on page 1), what is the numerical value of the coefficient of friction $\mu_k$ so that he just barely reaches the top of the second ramp?

Present your reasoning. Briefly discuss whether your numerical value seems “big” or “small” or “reasonable” for the real-life situation of Adam on a ramp.