Introduction: The Greek philosopher Aristotle believed that objects fall toward earth at a rate proportional to their weight. (Heavy things fall faster !?) Galileo Galilei found this to be absurd and not only proved it to be wrong through experimentation (thereby ushering in the era of experimental physics) but also developed the kinematical equations we now use to explain the motion of objects in “free fall”. Scientific lore has it that in 1589 while a young math professor at the University of Pisa, Galileo dropped two balls of the same size but different weights off the Leaning Tower of Pisa to demonstrate Aristotle’s error. Few historians believe this particular experiment was actually performed, but, if you’re interested, more information about Galileo and some of the experiments he did perform involving projectiles, inclined planes, and pendula can be found at: http://www.pbs.org/wgbh/nova/pisa/galileo.html

In an attempt to reproduce Galileo’s legendary experiment, a physics student leans out a window on the top floor of the Gamow Tower (the Physics building) at an elevation of 36 m above the sidewalk, waits to insure that there are no passers by, and drops two balls of the same size. “Drop” means that the initial downward velocity of each ball is zero. One ball is 0.8 kg and made of iron and the other is 1.2 kg and made of lead. Not only do the two balls hit the ground at the same time, the heights of the two balls match all the way down.

This result is an example of a general principle: An object in free fall near the earth’s surface has a downward acceleration of about \( g = 9.8 \text{ m/s}^2 \). “Free fall” means that only the force of gravity is acting on the object and, thus, air resistance can be neglected.

In this entire homework, assume that the coordinate system is as shown in the accompanying picture where the origin is at the base of the tower and positive is upward.
1. Which of the following graphs best represents the balls’ velocity versus time?

(a) \[ \text{Graph (a)} \]

(b) \[ \text{Graph (b)} \]

(c) \[ \text{Graph (c)} \]

(d) \[ \text{Graph (d)} \]

(If you think the answer is “none of these”, sketch what you think it should be and explain briefly)

2. Below, make a simple sketch (in the style of the answer choices above, but put a numerical scale on your vertical axis please) that represents the balls’ acceleration versus time.
3. For the time being (here and in #4), ignore the numerical values given above and replace them with variables. Let the height above the ground where the student releases the balls be $H$, $v_0$ be the initial velocity of each ball which we assume to be zero, $g$ be the magnitude of the acceleration of gravity, $v$ be the instantaneous speed of the ball at time $t$, $y$ be the instantaneous height of the ball at time $t$.

Which of the following expressions will give the time it will take for each of the balls to hit the ground? **Present your reasoning.** (If you prefer something else – what do you get?)

(a) $\sqrt{\frac{2gH}{v}}$  
(b) $\frac{g}{v}$  
(c) $\sqrt{\frac{2H}{g}}$  
(d) $\sqrt{\frac{H}{g}}$  
(e) $\sqrt{gH}$  
(f) Something else!

4. Again, ignore the numerical values given at the beginning of the assignment and replace them with the variables defined in #3. Construct a formula (in the style of the answer choices above) for the speed that the balls will be going when they hit the ground? **Present your reasoning.**

4b. Recall that last week’s written homework asked you to work out the dimensions of some equations. Explicitly show us that the dimensions of the equations you just got in problems 3 and 4 above come out to be what they should be (namely, a time in q3, and a speed in q4)
5. Now, recall the numerical values presented in the problem statement at the beginning of the assignment. How long after the balls are dropped will it take for them to hit the ground? Present your reasoning. (Hint: your solution to #3 may be useful.)

6. With the numerical values presented in the problem statement at the beginning of the assignment, at what speed will the balls be going when they hit the ground? Select the best choice, and present your reasoning. (Hint: your solution to #4 may be useful.)
(a) 26.6 m/s (b) 21.6 m/s (c) 18.8 m/s (d) 2.71 m/s (e) 7.35 m/s (f) None of these

7. Which of the following expressions gives the change in velocity of the balls between times $t_1$ and $t_2$? Remember, as in earlier problems, $g$ represents the magnitude of the acceleration of gravity. Present your reasoning.
(a) $g(t_1 + t_2)$ (b) $\frac{1}{2}g(t_2 - t_1)$ (c) $\frac{1}{2}g(t_1 - t_2)$ (d) $g(t_1 - t_2)$ (e) $g(t_2 - t_1)$ (f) None of these.
8. How does the change in velocity of the balls from \( t_0 = 0 \) to \( t_1 = 1 \) sec compare with the change in velocity from \( t_1 = 1 \) sec to \( t_2 = 2 \) sec. **Present your reasoning.**

(a) It is less.  
(b) It is the same.  
(c) It is greater.  
(d) It depends on the object.

9. An object in free fall falls (from rest) a distance \( \Delta y \) during the first \( t \) seconds. How far does it fall during the first \( 3t \) seconds? **Present your reasoning.**

(a) \( \Delta y + 3 \)  
(b) \( 3\Delta y \)  
(c) \( \Delta y + 9 \)  
(d) \( 9\Delta y \)  
(e) \( 4\Delta y \)  
(f) Something else!

10. A styrofoam ball of the same size as the lead ball takes a longer time to reach the ground. Which of the following is a good explanation for this? **Briefly, explain your reasoning**

(a) The force of gravity does not act on the Styrofoam ball. 
(b) The force of gravity on the Styrofoam ball is less than that on the lead ball. 
(c) Air resistance is a significant force in this problem. 
(d) There is a gravitational force between the ball and the building. 
(e) The student did not release the Styrofoam ball correctly. 
(I claim the answer is not “none of these”, one of them really does best explain the reason!)