Physics 1140 Laboratory

Name printed: _________________________________

Spring 2003

My Lab Section Number: __________

Homework Set #2

This homework is due at 5:00 p.m., Wednesday, February 5. Place it in your Section’s slot in the cabinet in the SE corner of Duane Room G2B66 (the lab). Please use these sheets of paper.

1. Chapter 3.7. Uncertainty in a function of one variable using derivatives: (3 pts)

Given that $t = 8.0 \pm 0.5$ s, what are the values of the functions below and their uncertainties? Please round uncertainties to one significant figure and round values according to uncertainties.

a) $z = t^4$

$z = 8^4 = 4096$ s$^4$

dz/dt = $4t^3 = 2048$ s$^3$

$\delta z = (dz/dt) \delta t = 2048 \times 0.5$ s$^4 = 1024$ s$^4$ rounds to 1000 s$^4$

$z \pm \delta z = (4000 \pm 1000)$ s$^4$

or $z \pm \delta z = (4 \pm 1) \times 10^3$ s$^4$

b) $z = 1/ t^2$

$z = 1 / 8^2 = 0.015625$ s$^{-2}$

dz/dt = $-2 / t^3 = 0.003906$ s$^{-3}$

$\delta z = (dz/dt) \delta t = 0.003906 \times 0.5$ s$^{-2} = 0.001953$ s$^{-2}$ rounds to 0.002 s$^{-2}$

$z \pm \delta z = (0.016 \pm 0.002)$ s$^{-2}$
2. More uncertainties: Find the uncertainty in the quantities below. (3 pts)

Use the step by step procedure, which for a) below, means applying the rules for differences to the difference in square bracket and then applying the rule for products to the product remaining.

a) \( z = \frac{w}{(y + x)} = \frac{(12 \pm 1)}{(25 \pm 3) + (10 \pm 1)} \)

\( s = y + x = 35 \)

\( z = \frac{12}{35} = 0.3429 \)

\( (\delta s)^2 = (3)^2 + (1)^2 = 10 \)

\( (\delta z/z)^2 = (1/12)^2 + 10/(35)^2 = 0.015 \)

\( \delta z/z = 0.123 \)

\( \delta z = 0.123 \times 0.3429 = 0.042 \) rounds to 0.04

\( z \pm \delta z = 0.34 \pm 0.04 \)

b) \( z = ye^{-2x} \) \( (y = 20 \pm 2, x = -2.0 \pm 0.1) \)

\( z = ye^{-2x} = (20 \pm 2) e^{(4 \pm 0.2)} = 1092 \)

\( dz/dx = y(-2e^{-2x}) = -40e^4 = 2184 \)

\( dz/dy = e^{-2x} = e^4 = 54.6 \)

\( (\delta z)^2 = [(dz/dx)\delta x]^2 + [(dz/dy)\delta y]^2 = [(2.706)0.1]^2 + [(0.135)2]^2 \)

\( (\delta z)^2 = 47524 + 10650 = 58174 \)

\( \delta z = 241 \) rounds to 200

\( z \pm \delta z = 1090 \pm 200 \)

\( z \pm \delta z = (1.09 \pm 0.2) \times 10^3 \)
3. **Uncertainty in trig functions:** If $x = 40^\circ \pm 4^\circ$ and $z = \cos [2 (x + 20^\circ)]$, then what is $\delta z$?

(2 pts)

Hint: Convert degrees to radians and use radian measure on your calculator. The derivatives of trig functions given in books and tables are written for angles in units of radians.

\[
\text{radians} = \text{degrees} \times \left(\frac{\pi}{180}\right)
\]

\[
2(x + 20^\circ) = 120^\circ
\]

\[
120^\circ \times \left(\frac{\pi}{180}\right) = \frac{2\pi}{3} \text{ radians}
\]

\[
z = \cos \left(\frac{2\pi}{3}\right) = -0.5
\]

\[
\delta x = \pi \left(\frac{4^\circ}{180^\circ}\right) = 0.0698
\]

\[
dz/dx = -2 \sin \left(\frac{2\pi}{3}\right) = -1.73
\]

\[
\delta z = (dz/dx)\delta x = 1.73 \times 0.0698 = 0.12 \text{ rounds to 0.1}
\]

\[
z \pm \delta z = -0.5 \pm 0.1
\]

4. **Uncertainty a function of two variables using derivatives:** (2 pts) An image is formed with a lens. If the object is found at distance $p$ and the image is found at distance $q$, then the focal length is $F = pq / (p + q)$. What is the algebraic expression for $\delta F$ in terms of $\delta p$ and $\delta q$?

\[
dF/dp = [q(p + q) - (1)pq]/(p + q)^2 = [q / (p + q)]^2
\]

\[
dF/dq = [p(p + q) - (1)pq]/(p + q)^2 = [p / (p + q)]^2
\]

\[
\delta F^2 = ((dF/dp)\delta p)^2 + ((dF/dq)\delta q)^2 = ([q / (p + q)]^2\delta p)^2 + ([p / (p + q)]^2\delta q)^2
\]

\[
\delta F^2 = [(q^2 \delta p)^2 + (p^2 \delta q)^2] / (p + q)^4
\]

\[
\delta F = SQRT\{[(q^2 \delta p)^2 + (p^2 \delta q)^2] / (p + q)^4}\}
\]

also

\[
\delta F^2 = [(\delta p / p^2)^2 + (\delta q / q^2)^2] \times [pq / (p + q)]^4
\]