Lab E3: The Wheatstone Bridge

Introduction

The Wheatstone bridge is used to compare an unknown resistance with a known resistance. The bridge is commonly used in control circuits. For example, a temperature sensor in an oven may have a resistance that increases with temperature. The control circuit should turn on the oven heater until the sensor in the oven has reached the desired resistance. The control knob (which may be labeled with temperature readings) adjusts a variable resistor to which the sensor is compared. The heater is turned on when the resistance is lower than the comparison value and turned off when it is higher.

![Schematic of a Wheatstone Bridge](image)

Fig. 1. Schematic of a Wheatstone Bridge

The unknown resistor is $R_x$, the resistor $R_k$ is known, and the two resistors $R_1$ and $R_2$ have a known ratio $R_2/R_1$, although their individual values may not be known. A galvanometer (a sensitive voltmeter) $G$ measures the voltage difference $V_{AB}$ between points A and B. Either the known resistor $R_k$ or the ratio $R_2/R_1$ is adjusted until the voltage difference $V_{AB}$ is zero and no current flows through $G$. When $V_{AB} = 0$, the bridge is said to be "balanced".

Since $V_{AB} = 0$, the voltage drop from C to A must equal the voltage drop from C to B, $V_{CA} = V_{CB}$. Likewise, we must have $V_{AD} = V_{BD}$. So we can write,

1. $I_a R_1 = I_b R_k$
2. $I_a R_2 = I_b R_x$

Dividing (2) by (1), we have

$$\frac{R_2}{R_1} = \frac{R_x}{R_k} \quad , \quad R_x = R_k \frac{R_2}{R_1}.$$

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Thus, the unknown resistance $R_x$ can be computed from the known resistance $R_k$ and the known ratio $R_2 / R_1$. Notice that the computed $R_x$ does not depend on the voltage $V_o$; hence, $V_o$ does not have to be very stable or well-known. Another advantage of the Wheatstone bridge is that, because it uses a null measurement, $(V_{AB} = 0)$, the galvanometer does not have to be calibrated.

In practice, the Wheatstone bridge is seldom used merely to determine the value of a resistor in the manner just described. Instead, it is usually used to measure small changes in $R_x$ due, for instance, to temperature changes or the motion of microscopic defects in the resistor. As an example, suppose $R_x = 10^6 \Omega$ and we wanted to measured a change in $R_x$ of $1\Omega$, resulting from a small temperature change. There is no ohmmeter which can reliably measure a change in resistance of 1 part in a million. However, the bridge can be set up so that $V_{AB} = 0$ when $R_x$ is exactly $10^6 \Omega$. Then any change in $R_x$, $\Delta R_x$, would result in a non-zero $V_{AB}$, which, as we show below, is proportional to $\Delta R_x$.

You would not weigh a cat by weighing a boat with and without a cat on board. So, you would not want to measure very small changes in $R_x$ by measuring $R_x$ with and without the change. Instead, you want to arrange things so that the change in $R_x$, $\Delta R_x$, is the entire signal. The bridge "balances out" the signal due $R_x$, leaving only the signal due to $\Delta R_x$. To show that $V_{AB} \propto \Delta R_x$, we consider Fig.1, and note that $V_{CD} = V_o$. We assume that the voltmeter has infinite resistance, so that no current flows through it, even when the bridge is not balanced. We also assume that the bridge has been balanced with the sample resistance at an initial value of $R_{xo}$, so that $R_{xo} = R_k(R_2 / R_1)$. Then we consider what happens to $V_{AB}$ when the sample resistor is changed by a small amount to a new value $R_x = R_{xo} + \Delta R_x$.

Applying Kirchhoff's Voltage Law and Ohm's Law to the upper and lower arms of the bridge, we have

$$V_o = I_a(R_1 + R_2) = I_b(R_k + R_x). \tag{4}$$

We are trying to find $V_{AB}$, which we can relate to $I_a$ and $I_b$.

$$V_{AB} = V_A - V_B = (V_C - V_B) - (V_C - V_A) = I_b R_k - I_a R_1 \tag{5}$$

We can use (4) to solve for $I_a$ and $I_b$ and then substitute for $I_a$ and $I_b$ in (5),

$$V_{AB} = V_o \frac{R_k}{R_k + R_x} - V_o \frac{R_1}{R_1 + R_2}. \tag{6}$$
This equation shows how $V_{AB}$ depends on $R_x$. Notice that eqn.(6) yields $V_{AB} = 0$ when $R_x = R_k \left( \frac{R_2}{R_1} \right)$. To see how much $V_{AB}$ changes when $R_x$ changes from $R_{xo}$ to $R_{xo} + \Delta R_x$, we write

$$
\Delta V_{AB} \approx \left( \frac{dV_{AB}}{dR_x} \right)_{R_x = R_{xo}} \cdot \Delta R_x = -V_0 \frac{R_k}{(R_k + R_{xo})^2} \cdot \Delta R_x.
$$

We wrote eqn.(7) by regarding $V_{AB}$ as a function of $R_x$, and remembering (from Calculus) that if $f = f(x)$, then $\delta f = \frac{df}{dx} \cdot \delta x$.

Substituting $R_{xo} = R_k \left( \frac{R_2}{R_1} \right)$ into (7) yields

$$
\Delta V_{AB} = \frac{-V_0 R_k}{R_k + R_k} \cdot \frac{R_2}{R_1} \cdot \Delta R_x = \frac{-V_0 R_2}{R_k \left( 1 + \frac{R_2}{R_1} \right)^2} \cdot \Delta R_x.
$$

Finally, we remember that $V_{AB,initial} = 0$, so the change in $V_{AB}$ is the same as $V_{AB}$, and we have

$$
V_{AB} = V_0 \frac{R_1}{R_1 + R_2} \frac{\Delta R_x}{R_k}.
$$

**Experiment**

We will use a slide-wire Wheatstone bridge (see figure on next page), in which the two resistors $R_1$ and $R_2$ are two portions of a single, uniform Ni-Cr wire. Electrical contact is made at some point along the wire by a sliding contact (this contact corresponds to point A). The two portions of the wire on either side of the contact have resistances $R_1$ and $R_2$, and the ratio $R_2 / R_1$ is the same as the ratio of the lengths of the two portions of wire, $L_2/L_1$. The lengths are readily measured with a meter stick which the wire rests upon.

The 10 $\Omega$ resistor in series with the 6 V power supply serves to limit the current through the bridge to less than 1A. (Higher currents might over-heat components of the bridge.)

Set the voltage from the power supply to approximately six volts with the “Coarse” voltage knob on the front panel of the supply. The voltage is read on the small voltmeter alongside the control knob.

The known resistor $R_k$ is adjustable and can be set to any value from 1 $\Omega$ to 999 $\Omega$ in 1 $\Omega$ steps with a decade resistance box, which is accurate to 0.02%. The box changes the resistance in units of 1, 10 and 100 ohms (look for the tiny labels on the knobs.) The Ni-Cr wire has a total resistance of about 2 $\Omega$. The sliding contact is spring loaded; you
have to push it down to make contact to the wire at point A. There are two buttons on the sliding contact; push one or the other, but not both, to contact the wire.

Fig. 2. Physical Layout of the Wheatstone Bridge.

Detail of the sliding wire contact:

Adjust the position of the sliding contact to balance the bridge (zero reading on the DMM which serves as our galvanometer).

For Part 1 of the lab, the unknown resistor $R_x$ is one of 5 coils of wire, numbered 1 through 5, mounted on a board. The lengths of the wires, their composition, and their gauge number are printed on the board. The gauge number is a measure of the thickness of the wire. 22 gauge wire has a diameter of 0.644mm; 28 gauge wire has a diameter of 0.321mm. For part 2 of the lab, $R_x$ is a stand-alone coil of copper.

The resistance $R$ of a wire is related to its length $L$, its cross-sectional area $A$, and its resistivity $\rho$ by the formula
The resistivity $\rho$ of a material depends on composition, on defects in crystal structure, and on the temperature. For metals, $\rho$ is approximately constant at very low temperatures ($T < 100$ K) and increases approximately linearly with temperature (measured in °K) at high temperatures.

At $T = 20$ °C, the resistivity of pure, defect-free, copper is $\rho = 1.678 \times 10^{-8}$ Ω m.

**Procedure**

**Part 1: Resistivity of Copper**

In this section, you will use the bridge to make precise measurements of the resistance of each of the 5 coils of wire on the coil board. First, however, use the digital multimeter (DMM) to make an approximate measurement of the resistance of each coil. You must temporarily disconnect the coil board and the decade box from the bridge when testing them with the DMM. With the DMM, measure the resistance of each of the five coils to the nearest 0.1 Ω. The resistance of the wire leads used to connect the DMM to the coils is not negligible, so first use the DMM to measure the resistance of the two wire leads in series (a few tenths of an ohm, typically), and then subtract this lead resistance from your measurements. **Check yourself:** the smallest coil resistance is below 1 ohm!

Now familiarize yourself with the decade resistance box. Adjust the knobs for 15 ohms and verify with the DMM that the value is indeed 15 ohms plus the resistance of the wire leads. For this measurement, disconnect the decade box from the bridge circuit and connect the DMM directly to the decade box.

Now use the bridge to measure the resistance of each of the 5 coils. Connect the DMM to the bridge as shown in Fig. 2. Set it to the most sensitive scale (400 mV). Before connecting the battery to the bridge, carefully check that all the connections are correct. Select a coil, attach it to the bridge, and set the decade box resistance $R_k$ to be as near to $R_x$ as possible (you know $R_x$ roughly from your DMM measurements). Balance the bridge by moving the sliding contact along the wire while watching the DMM. With the bridge balanced, measure $L_1$ and $L_2$, and compute

$$R = \rho \frac{L}{A}.$$
\[ R_x = R_k \frac{R_2}{R_1} = R_k \frac{L_2}{L_1}. \] (11)

Repeat this procedure for the other 4 coils. Be careful that you do not include more than one coil in the circuit.

For each of these coils, compute the resistivity, using your measured resistances, the data printed on the coil board, and eq'n (10). Make a table (or arrange columns side by side) with the headings: coil #, R from DMM, R from bridge, and computed resistivity. Four of the 5 sample coils are made of copper (Cu) \[ \text{Make sure that the coil data and resistance data are matched up correctly for the different coils} \]. For the four Cu coils, compute the average resistivity, \( \rho_{\text{Cu}} \) and its uncertainty \( \delta \rho_{\text{Cu}} \). Compare your average value with the known value of \( \rho_{\text{Cu}} \). Do they agree? Coil #5 is made of a copper-nickel alloy. What is the ratio \( \rho_{\text{Cu-Ni}}/\rho_{\text{Cu}} \)?

**Part 2. Temperature dependence of resistivity.**

For this part, the unknown \( R_x \) is a copper wire coil (the one which is not attached to the coil board). We will use eq'n (9) to measure the change in resistance of the coil when its temperature is changed by plunging it into an ice bath. Note that eq'n (9) can be written as

\[ V_{AB} = V_o \left( \frac{L_1}{L_1 + L_2} \right)^2 \frac{\Delta R_x}{R_k}. \] (9')

Prepare an ice bath by filling a beaker with ice from the icemaker (next to the video area) and adding some water. Measure room temperature and the ice bath temperature with the digital thermometer.

Measure the coil resistance \( R_x \) with the DMM and set \( R_k \) equal to \( R_x \), as nearly as possible. With \( R_x \) and \( R_k \) connected in the bridge, measure \( V_o = V_{CD} \) with the DMM. See Fig. 2 \( \text{(not Fig. 1)} \) to locate \( V_{CD} \). Note that \( V_{CD} \) is not the battery voltage. For the bridge measurement, again use the DMM set to the most sensitive range (400mV).

With the sample coil at room temperature, balance the bridge and compute the coil's resistance at room temperature. Now without changing the position of the slide wire contact, place the coil into the ice bath and watch as the voltage \( V_{AB} \) on the DMM changes. Record \( V_{AB} \) after it reaches equilibrium. Use eq'n (9) to compute \( \Delta R_x \), the change in resistance of the coil.

If it is true that resistivity \( \rho \) is proportional to the temperature, then the resistance \( R \) of the coil as a function of temperature can be expressed as \( R(T) = R_0 \left[ T \right. \left. (293 \text{ K}) \right] \), where \( R_0 \) is the room temperature resistance, \( T \) is the temperature of the coil, and 293 K is room temperature. Test this model by comparing your measured change in resistance to the change calculated from the model. Why should \( R \) increase with \( T \)?
Questions:  

Name_______________________________Section___________  
use other side if necessary

1. Two copper wires, labeled A and B, are at the same temperature, but the temperature is unknown. Wire B is twice as long and has half the diameter of wire A. Compute the ratio $R_B/R_A$.

2. A 28 gauge copper wire is 30 meters long. What is its resistance at room temperature?

3. Name two advantages of a Wheatstone bridge over an ordinary ohmmeter.

4. What is the formula for the total resistance of two resistors in parallel? Consider two resistors $R_1 = 3 \, \Omega$ and $R_2 = 200 \, \Omega$. What is the total resistance of these two resistors in parallel. Give your answer to the nearest ohm.
Questions:  

5. Consider Fig.2 and recall that the total resistance of the Ni-Cr wire is 2 Ω. If $R_k$ and $R_x$ are both very, very large compared to 2 Ω, how much current flows through the Ni-Cr wire?

6. Suppose an unknown resistance $R_x$ is measured with the bridge circuit shown in Fig. 2 and the result is $R_x = 8.65$ Ω. The 6 V battery is then replaced with an 10 V battery and $R_x$ is re-measured. What is the new measured value of $R_x$?

7. A sample of copper wire is 100 m long and has a diameter of 0.250 mm. Its resistance is determined to be 38.0 Ω. What is the resistivity of the copper in this wire?

8. (Counts as 3 questions) The Wheatstone bridge is often used in precision ovens and refrigerators to measure small changes in temperature. A Wheatstone bridge such as in Fig. 1 has a $V_o = 10.0$ V. With $R_k = 7.00$ Ω and the unknown resistor $R_x$ at room temperature, the bridge is balanced with $L_1 = 44.0$ cm and $L_2 = 56.0$ cm.

   a) What is the value of $R_x$?

   b) While still attached to the bridge, the unknown $R_x$ is placed in an oven, which raises its temperature by 23.0° C. The value of $V_{AB}$ is then found to be 0.087 V. What is the resistance change of the unknown, $\Delta R_x$?