Electric Currents and Resistance

So far, we’ve considered electrostatics, charges which (pretty much) stay put. In the demos of sparking Van de Graaf’s, or discharging capacitors, we’ve seen the (important) effects of charges moving, which leads us to discuss the flow of charges: electric currents.

Electric Currents:
Whenever charges are free to move (e.g. in conductors), if you apply an \( E \) field, they will move. (After all, \( F = qE \Rightarrow \text{acceleration!} \))
Imagine a wire, pick some spot, and ask yourself

\[
\text{How much charge passes by that spot each second? That’s the current.}
\]

Mathematically, current is called \( I = \frac{\Delta Q}{\Delta t} = \text{amount of charge/sec.} \)
Often written \( \text{d}Q/\text{d}t \), or even (more sloppily?) \( Q/t \) for short.
The units of current are Coulombs/sec = C/s = 1 Ampere = 1 A.
So if \( I = 1 \text{ A} \), that means 1 Coulomb flows by each second. (A LOT!)

Your choice of that little "spot" is quite arbitrary: If you moved it to the left, or the right, there's no difference. The SAME NUMBER of charges pass by each second. Even if you tilted and stretched that "area", making it bigger, the total current flowing past it would still be the same. (Think about that, convince yourself! We're just counting charges flowing by.

Current has a direction. If the current is to the right, there’s a net flow of charge to the right. This could occur in one of two ways:

It could mean “+”’s physically moving to the right

\[
\text{OR it could mean “-”’s physically moving to the left.}
\]

There’s (almost) no difference, in terms of “flow of charge”. Think about this, it’s an important point. Negatives moving left are in most ways equivalent to positives moving right. The flow of charge is the same in either case.
Here’s another way to think about this. Start with two neutral plates. Now, you could EITHER move some “-” charges down, OR move some “+” charges up, but either way, the final situation is the same.

Our convention is always to define current I as the flow of imaginary “+” charges. (Even though in reality it’s really negatives going the other way. In most conductors, it really is negative electrons flowing opposite the "conventional" current.)

What makes currents flow? Generally, electric fields make charges move. You can also think of it as arising from changes in electric potential energy: a change in potential energy means you can convert potential energy into kinetic energy (motion), like a hill makes water flow down it... We'll talk more about this next chapter, introducing batteries, and the more generic term "EMF"...

IMPORTANT: Current is NOT the same as voltage - not even close. Current is the flow of electric charges. Voltage is the energy per charge. Totally different! (Your primary intellectual task for the next 2 chapters is to create a clear mental model that distinguishes these things for yourself)

Some important concepts to be aware of:

1) In “ideal wires”, electrons are free to roam around. In good (perfectly) conducting metal it takes zero work to move electrons around. Metals like to be at an equipotential throughout, if they can. [There is no voltage drop along ideal wires.]

2) Charge is conserved. In steady state circuits, that means there is no buildup of charge anywhere. Whatever charge comes in to some point must go right on by, and out the other side.
This leads us to the question of how to represent current in diagrams. We generally just draw curvy lines with arrows, to indicate the direction of (conventional) current flow. The arrows follow wires (usually) It's a little funny though - mathematically, current as we defined it is NOT a vector, it's a number (positive means current is flowing in the direction shown. Negative means current is actually flowing the other way) This can be confusing at first! Let's do some examples: just remember, the bottom line of current conservation is: "whatever current goes in, must come out". (Think about why!)

At this junction "a" (which might be part of a bigger circuit),
\[ I_1 = I_2 + I_3 \]
Current Entering = Current Exiting

Just be careful to watch the arrows. You will be drawing arrows for currents, and they might point either way. E.g. in this picture:
\[ I_1 + I_3 = I_2 \]
Current Entering = Current Exiting
(Note the direction of I3's arrow)

**Fancier example:** We haven't learned all the symbols in this diagram, but never mind: it shows a battery, some wires, and resisters. Focus your attention at the junction labeled "a". Here,

\[ I \text{ (in)} = I_1 + I_2 + I_3 \]
(I enters, the other three exit)

What goes in must go out.

At the bottom of this circuit, I1, I2, and I3 all flow back together at node b, where we thus learn that the current flowing back into the bottom of the battery must be
\[ I \text{ (out)} = I_1 + I_2 + I_3 \]

That's the exact same as was flowing in at the top!

**Current is NEVER "eaten up", it just "flows" around circuits!**
Understanding current microscopically: current density:

Let's picture a chunk of metal (a wire) with charges drifting along in it. The chunk has cross section A, and the charges "q" each drift to the right with, on average, constant "drift" velocity to the right, \( v_d \).

Since \( I = \Delta q/\Delta t \), we must estimate how many charges pass through the area "A" in a given time. Think about this for a second: in time "\( \Delta t \)" , ANY charge \( q \) which started out in the dashed volume shown, that extends back a distance \( \Delta L = v_d \Delta t \) will make it past the area "A".

Do you see that? They're all drifting right. We want to know how MANY will pass "A" in time \( \Delta t \).

The ones that started a distance \( v_d \Delta t \) back will JUST make it. All the other ones that started inside that dashed volume certainly pass too!

Let's define \( n = "number of charge carries per volume" \)

\( = \) the "volume density of free charges".

That dashed cube contains \((#/volume)\times(volume)=(n) \) \( (dL\times A) \) charges. As we argued, they ALL make it past area A in time \( \Delta t \)

Thus, the total current

\( I = \Delta q/\Delta t = (# \text{ charges passing by})\times(\text{the charge on each one})/\Delta t \)

\( I = (n \; dL \; A) \times (q) / \Delta t = (n \; v_d \; \Delta t \; A) \times (q) / \Delta t = n \; v_d \; A \; q. \)

We define the current density \( \underline{J} = "current per unit area" = I/A, \)

so \( \underline{J} = n \; v_d \; q \)

Recap: current density \( \underline{J} \) tells you the # of charges flowing past some area each second, DIVIDED by the area being crossed. (It's a current DENSITY, not the current itself)

Most books define \( \underline{J} \) as a vector, \( \underline{J} = n \; q \; \underline{v}_d \) (This is useful especially if current is not confined in a wire. For us, we won't worry much about the vector aspect)
Comments: $v_d$ is not that big in wires. In fact, it's ZERO if there is no external electric field to push things along. It is NOT the same as the "random velocity" of the charges, which can be very large. It's the "drift" velocity, which tells you about the overall tendency to move in a specific direction.

If $q$ is negative (which is often the case), then the current flows opposite the direction the negatives move. This is correct, and makes sense: we talked about it right at the start of this chapter. (Think about it!) So if you have a fluid with both negative AND positive charge carries, like salty water, the positive might move right (causing a conventional current $n_1 q_1 v_1$, and the negatives might move left (causing a conventional current $n_2 |q_2| v_2$, ALSO to the right!!) And so the total current would be large, the sum of these two!

**A model:**

There is a simple microscopic picture of what's going on in metals: as you apply an external $E$ field, you might expect free charges to accelerate, going faster and faster, thus producing more and more current. But no, the charges in real metals are not completely free to move, they experience something very much like "friction". The faster they move, the more friction they feel (from bumping into imperfections, ions, other electrons...)

It's sort of like air drag, we can model the friction force $F_f = \eta v$.
($\eta$, pronounced "eta", is a coefficient that tells you how strong drag is)

So a free body diagram for a charge "$q$" inside a metal, in an external electric field $E$ to the right, would look like this:

\[ F_f = \eta v \quad F = qE \]

The charge will accelerate at first, but as $v$ gets bigger, $F_f$ gets bigger, until finally the forces balance, and then it stops accelerating.

It has reached a terminal velocity $v$, when $qE - \eta v = 0$

Thus, terminal velocity $v = (q/\eta)E$

(The bigger $E$, *or* the more charge you have, the higher $v$ is.
The bigger $\eta$ is, i.e. the more friction you have, the smaller $v$ is.
Makes sense)

Now, recall that $J = n q v$, and $v = (q/\eta) E$, so we have

$J = (n q^2/\eta) E$. We summarize this as

$J = \sigma E$. 

\[ \mathbf{J} = \sigma \mathbf{E} \] is a useful formula, but this is not a "law of nature"
It is only true for materials that fit our simple model.
These are called "Ohmic" materials.
Most metals obey this law, approximately but to very high accuracy.
The constant \( \sigma \) is called the "conductivity".
If you have a big \( \sigma \), then a given \( \mathbf{E} \) field will make lots of current!
That's what happens in a good conductor - hence the name!

Sometimes people invert the equation and rename the constants:
\[ \mathbf{E} = \left( \frac{1}{\sigma} \right) \mathbf{J} = \rho \mathbf{J}. \]
The new constant \( \rho = 1/\sigma \) is called the "resistivity".
The more "resistivity" you have, the LESS current will flow for a given field. Again, a fine name.

Note that \( \rho \) and/or \( \sigma \) are constants. They depend on the material, not the size, or shape of the chunk. Also note they have NOTHING WHATSOEVER to do with "charge density" or "surface charge density", which used those same symbols two chapters ago.
In our simple model, \( \sigma = (n q^2/\eta) \), so measuring the conductivity tells you something about the microscopic physics - what kind of charges are moving around, with what kind of friction.

The units of \( \rho \) are \([\mathbf{E/J}] = \text{[V/m]} / \text{[A/m^2]} = \text{[V m/A]}\)
We define one Ohm = 1 \( \Omega = 1 \text{ V/A} \),
so \( \rho \) has units "Ohm meters", \( \Omega \cdot \text{m} \)

For a superconductor, \( \sigma \) is infinite, \( \rho \) is 0.
For an insulator, \( \sigma = 0 \) (or nearly so) \( \rho \) is infinite (or huge)
For a "non-ohmic" material, \( \sigma \) depends on \( \mathbf{E} \), it's not a constant!
For conductors, it depends, e.g.
\( \rho(\text{Cu = Copper}) = 1.7E-8 \Omega \cdot \text{m} \)
\( \rho(\text{pure H2O}) = 2.6E5 \Omega \cdot \text{m} \)
\( \rho(\text{salty sea water}) = .22 \Omega \cdot \text{m} \)
\( \rho(\text{glass}) = 1E10 \text{ to } 1E14 \Omega \cdot \text{m} \)
\( J \) and \( E \) are hard to measure. They're, in a sense, "microscopic". You don't really know how many charge carriers pass by per unit area, you just measure how many TOTAL pass by. (That's easy, that's the current! Just by an ammeter, $30 at McGuckins!)

Similarly, \( E \) field is rather hard to measure. But voltage differences \( V \) are easy - a flick of the switch turns your ammeter into a voltmeter!

So let's take our microscopic results and recast them in terms of things that are easy to measure in the lab.

Suppose we have a chunk of metal, length \( L \), area \( A \), and we apply a uniform \( E \) field across it.

We know (from 2 chapters ago) that \( V = E \cdot L \) (if \( E \) is uniform)

(or, to be more careful, what I really mean is \( \Delta V = V_A - V_B = E \cdot L \))

So \( J = \frac{E}{\rho} \) is the same thing as \( \frac{I}{A} = \frac{(\Delta V/L) \cdot (1/\rho)}{ \cdot I} \)

Or, regrouping, \( \Delta V = (\rho \cdot L/A) \cdot I \)

We define this combination of constants \( R = (\rho \cdot L/A) \), getting

\[ \Delta V = IR \]

This formula is called "Ohm's Law".

Many people drop the "\( \Delta \)" and just write \( V = IR \).

Be careful, THINK about what "\( V \)" means then - it's the difference of voltage across the material, or the "voltage drop".

\( R \) is called the "resistance" (NOT the resistivity!)

Ohm's law is VERY useful, but is not like Newton's law, or Gauss' law. (It is only true for ordinary "ohmic" materials)

It is the exact same thing as \( J = \sigma \cdot E \), just written in terms of more easily measured quantities.

Units: \([R] = \text{volts/amps} = \frac{V}{A} = \text{Ohm} = \Omega \) (Greek Omega).

Note: \( 1\Omega = 1 \frac{V}{A} = 1 \frac{(J/C)}{(1 \text{ C/s})} = 1 \frac{J \cdot s}{C^2}, \text{ yikes!} \)
R = (ρ L/A) depends on the material (ρ) but ALSO the shape of the material (L and A).

Short or fat wires have smaller resistance.

Long, or skinny wires have larger resistance.

The area dependence can be a little confusing to some people. I like to think of water flowing through pipes. Big wide pipes offer lots of area for the water to spread out, i.e. low resistance to flow. Narrow pipes are harder to force water through. (And if there’s friction, the longer the pipe, the more resistance there is...)

Empirically, you find $R \propto L$ (longer wire => more resistance) and $R \propto 1/A$ (fat wires, like fat pipes, have smaller resistance)

This is all entirely consistent with $R = \frac{\rho L}{A}$ which we started from. Remember, ρ is NOT the density (same symbol, different physics) You have to look it up in tables. It depends on the material, but NOT the shape - that’s been factored out.

Typical metals have $\rho = 1E-8 \text{ Ohm}\cdot\text{m}$ or so. Thus, typical R of a normal sized Cu wire is tiny, $<< 1\Omega$

R across a person (from dry hand to dry hand) might be $.1 \text{ M}\Omega$

You can talk about R of lots of things: a tank of water, a plasma (remember the grape in the oven? The plasma ball was low R)

*Example*: Some Cu wire in class demos has 2 mm diameter. Suppose I cut a 2 m long piece (about 6 ft.) What is the resistance of this wire?

*Answer*: Look up the resistivity for copper: $\rho = 2E-8 \text{ Ohm}\cdot\text{m}$. That’s a property of the metal, no matter what the shape. And remember, area = $\pi r^2 = \pi (\text{diam}/2)^2$.

$R = \frac{\rho L}{A} = 2E-8 \text{ Ohm}\cdot\text{m} (2 \text{ m}) / \pi (1E-3 \text{ m})^2 = 0.01 \Omega$ (pretty small.)

(Notice how the units worked out too - resistivity has *different* units than resistance.)
Real wires do have small resistances. They’re almost ideal. Any real-life chunk of normal material, including metal, will thus resist the flow of electric charge. We call any such material “resistors”, and use this symbol in diagrams:

What does this mean? Electrons can flow through resistors, but not with quite the ease of an ideal wire. They will lose a little energy along the way. It doesn’t mean electrons get “eaten up” (charge is conserved, electrons cannot disappear), it just means they give up some energy.

Let's think more about Ohm's law, V = IR. Doubling the voltage drop will double the current flow, i.e. ΔV ∝ I.

The smaller R is, the less ΔV you need for a given current. Or equivalently, the smaller R is, the more current you’ll get for a given voltage drop. (Ideal wires have R=0, so ΔV=0 along them, as we’ve said before.)

In terms of personal safety, what hurts you, current or voltage? The answer is somewhat ambiguous. (Also depends on how "Ohmic" you are, which in turn depends on things like where and how the current can enter your body, and whether it's time independent or not)

Generally, 10 mA passing through your heart will kill you. (That's 1E-2 C/sec, which, divided by 1.6E-19 C/electron implies 6E16 electrons/sec) We'll talk more about this (important!) issue again!

Consider a 12 V car battery. Between your two hands there might be typically, a resistance of .1 MOhm. So, if you grab the two poles of a car battery, a current I = V/R = 12 V/1E5 Ohm = .1 mA will flow through your body. Turns out to be too small to notice. But with wet hands, if your resistance is down by a factor of 10, 1 mA might be noticeable. If you dropped a screwdriver across the poles (DON'T TRY THIS!) the resistance could be a fraction of an Ohm, and HUGE currents would flow. You will get sparks, possibly explosions. People get hurt like this!
Example: Suppose my bike light has two “AA” 1.5 V batteries in series, like this, and I measure the current (with a meter) to be 2 A.

What is the resistance, R, of the bulb?

Answer: We must first decide what the voltage drop V is across the bulb. (Is it 0 V? 1.5 V? 3 V?)

The total voltage difference, from top to bottom, is 3 V.

Can you see this? A “1.5 V” battery doesn’t mean that the end labeled “+” is at 1.5 V. It merely means that the + end is 1.5 V HIGHER than the - end.

That means the voltage drop from the top of the bulb to the bottom is 3 V, i.e. there are 3 V across the bulb.

\[ V = IR \text{ means } R = V/I, \text{ or } R(\text{bulb}) = 3 \text{ V} / 2 \text{ A} = 12.5 \text{ Ohm}. \]

Note: If you stacked one battery upside down in the above circuit, the bulb won’t light up. Can you see why? In this case, \( \Delta V(\text{total}) = +1.5 - 1.5 = 0 \text{ V}. \)

There is no voltage drop across the bulb, no current flows.

(This is why you have to follow the little picture for “battery orientation” in your flashlight, or the bulb won’t light up) More on batteries and circuits next chapter!
**Power and energy in circuits:**

When current flows through a resistor, electrons are bumping into atoms (like skiers hitting the moguls): it’s like friction, energy is getting dissipated. (Electrical PE is being converted into heat)

Recall our definition of power, energy converted per sec: \( P = \frac{\Delta E}{\Delta t} \). Unit of power is J/s = Watt = W.

Now think of current running through a resistor. There is a voltage drop “V” across the resistor. Remember what “voltage drop” means: voltage drop is the change in electrical energy per unit charge: \( \Delta V = \frac{\Delta \text{Energy}}{Q} \)

Since voltage drop is just called V here, then the energy drop is \( Q \cdot V \), and power dissipated = \( \frac{\Delta \text{Energy}}{\text{time}} = QV/t = (Q/t) \cdot V = I \cdot V \).

This is a very important result: \( P = IV \),

Power = current * (change in) voltage.

• Don’t forget we keep dropping the \( \Delta \), what we mean by V in that formula is the change in voltage.

For normal resistors, Ohm’s laws says \( V = IR \) (or, \( I = V/R \)). That means we could rewrite \( P = IV = I^2R = \frac{V^2}{R} \).

(All equivalent and correct, for “Ohm’s law” resistors. For any other system, you should just use the fundamental formula \( P = IV \))

Which of those 3 forms (\( IV, \ I^2R, \text{ or } \frac{V^2}{R} \)) should you use when solving problems? It depends on what’s given, and perhaps what is constant.

E.g. if a battery is attached across a resistor, V is constant and given, so it’s probably most practical to use \( P = \frac{V^2}{R} \). But you have to look at each problem and think about it, there’s no general rule...
Example: A 100 W bulb is plugged into a 120 V wall socket. What’s R of the bulb? What is the current I flowing through the bulb?

Answer: There are many ways to go about this. Here’s one:

\[ P = \frac{V^2}{R}, \text{ so } R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{100 \text{ W}} = 144 \text{ Ohm} \]

Unit check: \[ \frac{V^2}{W} = \frac{V^2}{(J/s)} = \frac{V*V}{(J/s)} = \frac{V*(J/C)}{(J/s)} = \frac{V}{(C/s)} = \frac{V}{A} = \text{Ohm!} \]

So it worked out. I admit - units can get a bit nasty in this business!

By the way, 144 Ohms is a lot. You can't make a bulb from copper. You would use W (Tungsten) (or perhaps a pickle): something with more resistance, so it draws lots of current, and thus power, and glows!

Next, \[ P = V*I \] tells us \[ I = \frac{P}{V} = \frac{100 \text{ W}}{120 \text{ V}} = 0.83 \text{ A} \].

Or, alternatively, \[ P = I^2 R \] says

\[ I = \sqrt{\frac{P}{R}} = \sqrt{\frac{100\text{ W}}{144 \text{ Ohm}}} = 0.83 \text{ A}, \]
gives us another little check on our answer...

By the way *I’ve been cheating a little in the last examples*, because the 120 V in your wall socket isn’t quite like a battery: \( V \) really oscillates in time. We'll talk about this later in the semester!
**Electric costs:**

The power company does not charge for power (!)
It sells you *energy*. Power = Energy/time, so
Energy = Power*time.

They don’t really care if you use a LOT of power for a short time,
or a little power all the time, they charge you for the product: energy.

My public service bill charges me about 10 cents/ (kW hr). 
That’s pretty cheap by national standards, by the way, and I’m paying
2 cents more than most people to get wind-power...

Those are really the units on my bill. A kW·hr is a unit of ENERGY,
1 kW·hr = 1000 W * 1 hr = 1000 J/s * (3600 sec/hr) = 3.6E6 J.
(10 cents for 4 Mega Joules of energy. It’s really stunningly cheap!!)

**Example:** You buy an electric space heater, rated at 1 kW.
(That’s a pretty typical power rating for a big appliance. A blow-dryer
might even be 2 kW) You leave it on in the basement, day and night,
from October - March, through the cold months.
How much have you paid, extra, on your electric bills?

**Answer:** Let’s convert 6 months into hours:
6 mo * (30 days/month) * (24 hrs/day) = 4300 hours.

You used energy E = P(ave)*time = 1 kW * 4300 hrs = 4300 kW·hrs.
Cost to you is about 4300 kW·hrs * 10 cents/kW·hr = $430. Yikes!

(There are more efficient ways of heating a space than electric heaters.
All-electric heat in houses is pricey and it’ll only get worse as energy
costs start climbing in the near future! Might be better to at least first
insulate the room well.)

E.g. if you look up the *pure* chemical energy content of gasoline, 4300
kW hrs is the amount of energy stored in about 100 gal of gas. At
$2/gal, the cost of gasoline would be cheaper by a factor of two. (But,
then you'd have to figure in any inefficiency of conversion...)
Gasoline in the USA is still an absurdly cheap source of energy. The
"hidden costs" (e.g. from environmental impact) are only beginning to
be understood, and if they were added to the price, we'd be paying
through the nose. (And we probably WILL be in the near future!)
Semiconductors and other non-ohmic materials.

If you think back to our model, "n", the volume density of charge carriers, is NOT a constant. (And then remember, \( \rho = \eta/n \ q^2 \))

It can vary with temperature, or incident light, or external applied E fields.... If "n" is not constant, then R is not constant, and that means it doesn't obey the simple Ohm's law.

In a semiconductor, if the temperature is low, n tends to get smaller, there are fewer "charge carriers" available, and so \( \rho \) gets bigger. (More resistive)

For a metal, it's just the opposite: at low temperatures the thermal vibrations decrease, which makes for LESS resistance, i.e. lower \( \eta \)

So the charges have an EASIER time flowing, \( \rho \) gets smaller. (even metals are not perfectly Ohmic, although if you keep the temperature fixed, that's not such a problem)

As Temp goes up, more thermal vibration increases \( \eta \), in fact, pretty linearly. Lots of modern thermometers work by using this fact:
measuring R of a material and comparing with the nominal (standard) value directly tells you the temperature!

\( \rho \) does not go to 0 as Temp \(-\) 0, except for superconductors, because there are always impurities to cause resistance.

From this graph, since I = V/R, at low temperatures you see that R is lower, and thus the current is higher. Light bulbs in the US are usually plugged into 120 V sockets. Since V=IR (and V is fixed at 120 V) then when the bulb is COLD (low R), I must be big. Later, after the bulb has warmed up (and bulbs heat up a lot!) R is larger, so the current flowing through it is smaller. Have you noticed that light bulbs usually burn out when you first turn them on? They're coldest then, so R is lowest: the current is (momentarily) largest, and P = IV is therefore also momentarily the largest: the big rush of current/power breaks the fragile filament. Once it’s warmed up, it’s much less likely to break (unless you jiggle it)