A spring-loaded gun fires a dart at an unknown angle $\theta$ above the horizontal. Assume no air resistance and no friction anywhere. The mass $m$, spring constant $k$ and the initial compression $x$ of the spring are all known. Can you solve for the maximum height $h$ above the starting point using Conservation of Energy?

A) Yes

B) No

To find $h$, we also need to know at least the x-component of the initial velocity.

$$PE_i + KE_i = PE_f + KE_f$$

$$\frac{1}{2} kx^2 + 0 = mgh + \frac{1}{2} mv_{0x}^2$$
Assignments

Announcements:
• Complete CAPA 8 and HW 8 next week.
• LA applications will close Oct 24: lacentral.colorado.edu
• Midterm 2 will be next Thursday. Will cover up through Ch. 8.2. with emphasis on Chs. 4-8.2.

Today:
• Two more examples of problems when we need to combine other information with Conservation of Energy.
• Introduce graphs of Energy vs position and their interpretation.
• Power.
General Statement of the Conservation of Energy

$$E_{\text{total}} = E_{\text{mechanical}} + E_{\text{thermal}} = \text{constant}$$

Caution: it’s important to remember that although thermal energy (TE) is produced as a physical system evolves, it cannot be converted back to mechanical energy (ME) directly. Therefore, ME can covert to TE but TE cannot convert to ME directly.

$$KE_i + PE_i = KE_f + PE_f + E_{\text{thermal}}$$

We place $E_{\text{thermal}}$ on the right-hand-side of the equation and consider it to be the thermal energy produced during the evolution of the system. It is the difference between the final thermal energy of the system and the initial thermal energy of the system.

Typically: $W_{\text{friction}} = -E_{\text{thermal}}$
Two more examples where it’s necessary or at least useful to introduce other information to the conservation of energy.
**Example 1.** A box of mass $M$ slides down a **rough** incline from height $h$. The box has a speed $v_0 = 0$ at the top of the incline and $v$ at the bottom. Find the speed at the bottom of the incline.

Another measure of the work done by friction:

$$W_{frict} = -\mu_k NL = -\mu_k (mg \cos \theta)L$$

$$= -\mu_k (mg \cos \theta) \frac{h}{\sin \theta} = -\mu_k mgh \cot \theta$$

$$= \frac{1}{2} mv^2 - mgh$$  \quad  \text{(from energy conservation)}$$

$$\therefore v = \sqrt{2gh(1 - \mu_k \cot \theta)}$$
Example 2. A small track, starting at rest, slides without friction along a frictionless rail in the shape of a loop. The maximum height of the track is the same as the initial height of the ball. 
ASSUME NO FRICTION.

Will the ball make it to the top of the loop?

Example of using the conservation of energy together with consideration of the existence of a normal force.
Will it have enough Mechanical Energy?  Yes.

Will it stay in contact with the track?

What will be the condition for the ball to stay in contact with the track?

A) $N = mg$  B) $N > 0$  C) $N < mg$  D) $N > mg$
Will it have enough Mechanical Energy? Yes.

Will it stay in contact with the track?

Stay in contact: \[ \sum F_R = ma_R \quad \text{Newton II} \]

\[ N + mg = \frac{mv^2}{r} \quad \Rightarrow \quad \frac{mv^2}{r} > mg \]

\[ \therefore v > \sqrt{gr} \quad \text{(where } r = h/2) \]
A small ball, starting at rest, slides without friction along a frictionless track in the shape of a loop. The maximum height of the track is the same as the initial height of the ball. **ASSUME NO FRICTION.**

Will the ball make it to the top of the loop?

A) Yes, the ball will make it to the top of the loop.

B) No, the ball will not make it to the top.

C) Not enough info to say.

It won’t have enough speed to stay in contact with the track.

**Homework problem:** How much higher $H$ than $h$ would it have to start from to actually make it all around the loop?
h gives us PE: can replace h with PE.
No friction:
\[ E_{\text{tot}} = ME = KE + PE \] is constant: horizontal line gives \( E_{\text{tot}} \).
Can read the PE & KE off the curve for every position.
A cart rolls without friction along a track. \( E_{\text{tot}} = ME = KE + PE = 45 \text{ kJ} \). The top of the hill at \( x = 62 \text{ m} \) is round and the bottom of the valley at \( x = 130 \text{ m} \) is flat.

To within 5 kJ, what is the maximum KE?

A) 25 kJ  
B) 7 kJ  
C) 45 kJ  
D) 35 kJ

\[ KE = E_{\text{tot}} - PE \]
\[ = 45 \text{ J} - 10 \text{ J} \]
\[ = 35 \text{ J} \]
A cart rolls without friction along a track. $E_{\text{tot}} = ME = KE + PE = 45 \text{ kJ}$. The top of the hill at $x = 62 \text{ m}$ is round and the bottom of the valley at $x = 130 \text{ m}$ is flat.

When KE is minimum (over this stretch of track), what is the direction of acceleration?

A) Up  
B) Down  
C) Left  
D) Right  
E) Other direction or zero.
The difference between walking and running up these stairs is power – the change in gravitational potential energy is the same.

Power is the rate at which work is done –

$$\bar{P} = \text{average power} = \frac{\text{work}}{\text{time}}$$

Units: 1 W = 1 J/s
Rate at which energy is produced or used: Power

\[ P = \frac{mgh}{\Delta t} \]

Student A: \( mg(10m) / 8s = 1.2mg \) W

Student B: \( mg(20m) / 16s = 1.2mg \) W

A) A
B) B
C) Tie

\[ \bar{P} = \text{average power} = \frac{\text{work}}{\text{time}} \]