If a wire loop moves into the field of a stationary magnet, a motional emf is created:

\[ E = -d\Phi/B \text{, where } \Phi = \oint \vec{B} \cdot d\vec{l} \]

But a stationary loop and a moving magnet is the same physical situation.

In both cases, Michael Faraday found experimentally that \( E = -d\Phi/B \) is the same for I, II. But in case II, the \( E \) can't be due to magnetic force per charge (\( F = \nabla \times \vec{B} = 0 \) since \( \vec{v} = 0 \)). In case II, \( E \) must be due to an electric field \( \vec{E} \).

**Faraday's Law**

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{S} \vec{B} \cdot d\vec{a} \]

Can use Stokes' Theorem to write in differential form:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

This is a new law of physics, discovered and verified experimentally. It can't be derived.
So now there are 2 ways to make an $\vec{E}$-field:

1) Changes create curl-free $\vec{E}$-fields ($\nabla \times \vec{E} = 0$)

2) Changing $\vec{B}$-fields create "curly" $\vec{E}$-fields ($\nabla \times \vec{E} = -\partial \vec{B} / \partial t$)

Example: $\vec{E}$ created by changing $\vec{B}$ in a solenoid.

Recall $\vec{B}$ of long solenoid:

\[ \vec{B} \text{ inside } = \mu_0 n I \text{ turns/length} \]

Suppose current $I$ is ramping up, so $\vec{B}$ is steadily increasing:

\[ \vec{B}(t) = C \cdot t \quad (C = \frac{dB}{dt} = (+) \text{ const}) \]

$\vec{E}$ must be in azimuthal direction only. It can't have radial component because $\nabla \cdot \vec{E} = \phi_\theta = 0$

No! Not allowed, because no $Q_{\text{enc}}$ anywhere, so must have $\oint \vec{E} \cdot d\vec{a} = 0$

OK. For $r < R$, $\oint \vec{E} \cdot d\vec{a} = 2\pi r \vec{E}$, and

\[ \iint \vec{B} \cdot d\vec{a} = \vec{B} \cdot \pi r^2 \text{ so...} \]
\[ \oint E \cdot dl = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a} \]

\[ E \cdot 2\pi r = -\frac{d}{dt} \int \frac{d\vec{B}}{dt} \cdot d\vec{a} \]

\[ |E(r)| = \pm \frac{r}{2} \cdot C \]

Don't worry about \pm sign. Just direction of \vec{E} from 

For \( r > R \)

\[ \oint E \cdot dl = -\frac{d}{dt} \int \frac{d\vec{B}}{dt} \cdot d\vec{a} \]

\[ E \cdot 2\pi r = -\frac{d}{dt} (\vec{B} \times R^2) \]

\[ \left| E \right| = \frac{R^2}{2r} \cdot \frac{d\vec{B}}{dt} = \frac{R^2 C}{2r} \]

\[
\begin{align*}
\text{Inductance} & \quad \text{Consider 2 loops of wire, 1 and 2, near each other. Loop 1 has current } I_1. \\

\text{Biot-Savart says} & \quad \vec{B}_1 = \frac{\mu_0 I_1}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = \vec{B}_1 \text{-field due to current } I_1. \\

\text{Notice } \vec{B}_1 \text{ is complicated function of position,} & \quad \text{but it has very simple dependence on } I_1: \\
\vec{B}_1 \propto I_1 \quad \text{(double } I_1, \text{ } \vec{B}_1 \text{ doubles everywhere}) \\
\text{Now, flux thru } \mathcal{L}_2 \text{ due to } \vec{B}_1 = \Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2
\end{align*}
\]
But \( B_1 \propto I_1 \), so \( \Phi_2 \propto I_1 \).

Define mutual inductance \( M \) as \( \Phi_2 = M_{21} \cdot I_1 \).

Similarly, if \( L_2 \) has current \( I_2 \), then this \( I_2 \) causes a flux through \( L_1 = \Phi_1 = M_{12} \cdot I_2 \).

Griffith shows that \( M_{12} = M_{21} = M \).

A transformer is a simple device (no moving parts) consisting of 2 coils of wire that have a mutual inductance. See notes "Transformers".

Also, a current in a coil makes a flux in itself.

\[
\Phi = \int_{S} B \cdot d\alpha
\]

\( \Phi = LI \), \( L = \) self-inductance of coil

"Inductor" = coil of wire

Faraday \( \Rightarrow \)

\[
\mathcal{E} = -\frac{d\Phi}{dt} = -L \frac{dI}{dt}
\]

If you change the current in an inductor, an emf ("back-emf") is created which opposes the change in flux, thus opposing the change in current. Inductors act like current regulators; they try to maintain constant \( I \).

The current in an inductor cannot change instantly.
If $I$ tries to change suddenly, $\frac{dI}{dt} \to \infty$, $E = -\mu \frac{dI}{dt} \to \infty$, and $E$ prevents sudden change.

Calculation of $L$ is very messy for a single loop because $B$ is complicated function of position.

\[ E = \oint \overrightarrow{B} \cdot d\overrightarrow{a} = L \cdot I \]

but $L$ is easy to compute for a long solenoid:

\[ L = \frac{N \cdot B \cdot A}{n} \]

\[ L = \mu_0 n^2 A \cdot L \]

units of $L$, \[ L = \frac{1}{A} \cdot m^2 = \text{Henry (H)} \]

units of $I$ are called henries. A coil of several turns w/dimensions of centimeters has an inductance in the range $10^{-6} \to 10^{-3}$ H.

A Henry is a huge inductance