INSTRUCTORS MANUAL
Line Integrals and Work

Goals:
Students should be able to:

• set up (i.e., parametrize) and compute a line integral for a simple vector function,

• discuss the differences (as they relate to line integrals) of conservative and non-conservative vector fields,

• sketch simple vector fields in 2 dimensions, and

• determine and defend their choice of path for conservative vector fields.

This tutorial is based on:

• Original work written by Danny Caballero, Alysia Marino, and Steve Pollock.

Materials needed:

• (optional) whiteboards, dry-erase markers, and erasers

Special Instructions:

• None.

Homework Connections: Homework 4, Problem X

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Reflections on this Tutorial:

Another version of this tutorial was given to students in which Part 1 was much different; it involved mostly making calculations. Part 2 was not given to students.

Students required 20 minutes to complete the first page because they were computing so many integrals; this has been trimmed in this version. Although, it took them a while, most students had little trouble performing the computations. About half the class had seen conservative fields before.

Students taking the old version were very interested in “short-cutting” or visualizing path integrals; hence, the second part of this tutorial should be quite useful.
Part 1 – Taking a Line Integral

Consider the vector field equation below:

\[ \vec{F} = y\hat{x} \]

i. Compute the line integral, \( \int \vec{F} \cdot d\vec{l} \), from \( \langle 0, 1 \rangle \) to \( \langle 1, 0 \rangle \) using two paths:

1. a straight line path from \( \langle 0, 1 \rangle \) to \( \langle 1, 0 \rangle \) and

2. a piece-wise path that travels down the +y-axis to the origin, then makes a 90° turn and travels along the +x-axis.

How do the results compare?
Consider the vector field equation below:

\[ \vec{F} = -x\hat{x} - y\hat{y} \]

ii. Sketch the vector field in the box below.

iii. Can you think of a physical situation that might produce this vector field?

iv. Choose a path to compute the line integral, \( \int \vec{F} \cdot d\vec{l} \), from \( \langle 0, 1 \rangle \) to \( \langle 1, 0 \rangle \). How did you choose your path? Does it matter which path you choose? Discuss with your group.

PAUSE, check your results with your instructor.
Part 2 – Choosing an appropriate path

The line integral between to points in a vector field that has no curl (i.e., $\nabla \times \vec{f} = \vec{0}$) is independent of the path you choose. This doesn’t mean that the line integral is zero, just that its value only depends on the location of the end points.

i. Below are plots of several vector fields which have no curl. Two points, labeled A and B, appear in each diagram. Sketch the best choice of path for the given field if you were asked to compute the line integral from A to B. **Do not actually compute the line integral.**

- $\vec{f} = \hat{x}$
  - A: $(-2, -2)$
  - B: $(1, 3)$

- $\vec{f} = y\hat{y}$
  - A: $(-2, 3)$
  - B: $(2, -1)$

- $\vec{f} = x\hat{x} + y\hat{y}$
  - A: $(-2, -2)$
  - B: $(0, \sqrt{8})$

- $\vec{f} = y\hat{x} + x\hat{y}$
  - A: $(-3, -3)$
  - B: $(3, -3)$
ii. What helped you determine the best choice of path? Discuss with your group members.

iii. What does this tell you about starting a problem involving a line integral?