Harmonically Forced SDOF Oscillator
Harmonically Force Driven SDOF Oscillator

(a) SOF system

\[ F = \text{m} \cos \Omega t \]

(b) DFBD

\[ F_s = k u \]
\[ F_d = c \dot{u} \]

\[ F_l = m \ddot{u} \]

Static equilibrium position

\[ u = u(t) \]

\[ p(t) = p_0 \cos \Omega t \]
Equation of Motion

From the Dynamic Free Body Diagram (DFBD) of previous slide, we get the EOM:

\[ m \ddot{u} + c \dot{u} + k u = p_0 \cos \Omega t \]

Damping, modeled by the \( c u \) term, is included from the start since it is important in finding the maximum amplification at resonance.

The EOM is \textbf{linear and second order ODE}, as in the previous Lecture, but now this ODE is \textbf{non-homogeneous}. According to the theory of such equations, the solution \( u(t) \), which is called the \textbf{displacement response} is the sum of \textbf{two components}.
Response Decomposition

As remarked in the last slide, the total response $u(t)$ can be expressed as the sum of two components: called homogeneous and particular in applied math textbooks:

$$u(t) = u_H(t) + u_P(t)$$

Engineers use a terminology with closer connection to physics:

- **homogeneous solution** $\Rightarrow$ transient response
- **particular solution** $\Rightarrow$ steady-state response
Sourse and Significance of Transient Vs. Steady Response

The transient (=homogeneous) response is the solution under zero force. It is primarily determined by initial conditions.

The steady-state (=particular) response is produced by the applied force.

If there is at least a tiny amount of damping, the transient solution decays at time $t$ grows, and eventually only the steady-state component survives. This explains the "transient" qualifier.
Steady Response Expression

See Lecture Notes
Magnification Factor and Phase Lag as Function of Frequency Ratio

Graph (a) shows the magnification factor $P_s$ as a function of frequency ratio $r = \Omega/\omega_n$ for various values of damping ratio $\xi$. Graph (b) shows the phase lag $\alpha$ in degrees as a function of frequency ratio $r = \Omega/\omega_n$ for the same range of $\xi$ values.
Magnification Factor and Phase Lag as Function of Frequency Ratio - Log-Log Plots

(a)

(b)

Frequency ratio $r = \Omega/\omega_n$

- $\xi=0.01$
- $\xi=0.1$
- $\xi=0.2$
- $\xi=0.5$
- $\xi=1$
- $\xi=5$

Magification factor $D_s$

Phase lag $\alpha$ in degrees

Frequency ratio $r = \Omega/\omega_n$
SDOF Oscillator Excited by Harmonic Base Motion

\[ F = m \ddot{u} \]

\[ F = k (u - u_b) \]

\[ F_d = c (\dot{u} - \dot{u}_b) \]

\[ F_l = m \ddot{u} \]

\[ u_b = U_b \cos (\Omega t - \alpha_b) \]

\[ u = u(t) \]

(a) Prescribed base motion

(b) Static equilibrium position for zero base motion