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Torsion Of
Closed Thin Wall
(CTW) Sections
Recall the Classification of Thin Wall (TW)
Cross Sections Under Torque

Closed Thin Wall (CTW) Sections
at least one cell shear flow circuit can be established
Single Cell: just one cell
Multicell: more than one cell

Open Thin Wall (OTW) Sections
no cell shear flow circuit can be established

Hybrid Thin Wall (HTW) Sections
contains both OTW and CTW components
Sample CTW Cross Sections

tube

box

aircraft fuselage

space shuttle fuselage

car frame (unibody construction)

wing torque box
**Single-Cell CTW Section**

(a) $t = t(s)$

(b) Shear flow $q = \frac{T}{2A_E}$

(c) Enclosed area $A_E$
Shear Stress Formula

Equilibrium of internal torque $T$ with the assumed shear stress distribution (constant across wall thickness, directed along tangent to midline) gives

$$T = \int dT = \int q \cdot h \, ds = q \int h \, ds = q \int 2 \, dA_E = q \cdot 2A_E = 2q \cdot A_E$$

in which $A_E$ is the *enclosed area* defined by the wall midline (which should NOT be confused with the *cross section area* or material area)

Solving for $q$ gives the shear flow and wall stress formula

$$q = \frac{T}{2A_E} \quad \tau = \frac{q}{t} = \frac{T}{2t \cdot A_E}$$

Since $q$ is constant along the contour, plainly the maximum shear stress occurs where the wall thickness $t$ is minimum:

$$\tau_{\text{max}} = \frac{q}{t_{\text{min}}} = \frac{T}{2t_{\text{min}}A_E}$$
Twist Angle Formula

It can be shown that the twist-angle rate is given by

\[
\phi' = \frac{d\phi}{dx} = \frac{q}{2GA_E} \int \frac{ds}{t} = \frac{T}{4GA_E^2} \int \frac{ds}{J} = \frac{T}{GJ}
\]

in which \( J = \frac{4A_E^2}{\int ds} \)

Here \( G \) is the shear modulus. Derivation of this formula requires the use of energy methods in mechanics, which is a graduate-level topic.

If the wall thickness \( t \) is constant along the cell contour, the foregoing formula simplifies to

\[
\phi' = \frac{d\phi}{dx} = \frac{q p}{2A_E t} = \frac{T}{GJ}
\]

in which \( J = \frac{4A_E^2 t}{p} \)

where \( p = \int ds \) is the midline perimeter length, or simply the perimeter.
Multicell CTW Sections
Are Not Treated in This Course

(a) y

(b) q_1

(c) q_2 - q_3

Cell flows

Wall flows
worked out example cross sections

(a) CTW section: rectangular tube

(b) OTW section: slitted rectangular tube

(c) Hybrid section: CTW + OTW (2)

All $d$'s are midline dimensions
Torqued Hybrid TW (HTW) "A" Section
For Recitation

(b)(a)

\[ b_1 = 200 \text{ mm}, \quad b_2 = b_3 = 400 \text{ mm}, \quad t_1 = t_2 = t_3 = 6 \text{ mm} \]
\[ G = 80 \text{ GPa (steel)} , \quad T = 604 \times 10^3 \text{ mm-N}, \quad L = 2514 \text{ mm} \]