Overview
Course Coverage

This course consists of three Parts:

I. *Finite Element Basic Concepts*

II. *Formulation of Finite Elements*

III. *Computer Implementation of FEM*
Where the Course Fits

The field of Mechanics can be subdivided into 3 major areas:

- Theoretical
- Applied
- Computational
Branches of *Computational Mechanics* can be distinguished according to the physical focus of attention.

**Computational Mechanics**

- Nano and Micromechanics
- Continuum Mechanics:
  - Solids and Structures
  - Fluids
  - Multiphysics
- Systems
Computational Solid and Structural Mechanics

A convenient subdivision of problems in Computational Solid and Structural Mechanics (CSM) is

\[ \text{Computational Solid and Structural Mechanics (CSM)} \]

\[ \begin{align*}
\text{Statics} \\
\text{Dynamics}
\end{align*} \]
CSM Statics

A further subdivision of problems in CSM Statics is

\[
\text{CSM Statics} \begin{cases} \\
\text{Linear} \\
\text{Nonlinear} \\
\end{cases}
\]
For the numerical simulation on the computer we must now choose a \textit{spatial discretization method}: 

- \textit{Finite Element Method}
- \textit{Finite Difference Method}
- \textit{Boundary Element Method}
- \textit{Finite Volume Method}
- \textit{Spectral Method}
- \textit{Mesh-Free Method}
Having selected the FEM for *discretization*, we must next pick a *formulation and a solution method*:

**Formulation of FEM Model**

- Displacement
  - Equilibrium
  - Mixed
  - Hybrid

**Solution of FEM Model**

- Stiffness
  - Flexibility
  - Mixed
Summarizing: This Course Covers

*Computational structural mechanics*

*Linear static problems*

*Spatially discretized by displacement-formulated FEM*

*Solved by the stiffness method*
What is a Finite Element?

Archimedes' problem (circa 250 B.C.): rectification of the circle as limit of inscribed regular polygons
Computing $\pi$ "by Archimedes FEM"

<table>
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<th>$n$</th>
<th>$\pi_n = n \sin(\pi/n)$</th>
<th>Extrapolated by Wynn-$\varepsilon$</th>
<th>Exact $\pi$ to 16 places</th>
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The Idealization Process for a Simple Structure

Introduction to FEM

Roof Truss

Physical Model

member

joint

support

IDEALIZATION & DISCRETIZATION

Mathematical and Discrete Model
# Two Interpretations of FEM for Teaching

<table>
<thead>
<tr>
<th><strong>Physical</strong></th>
<th><strong>Mathematical</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakdown of structural system into components (elements) and reconstruction by the assembly process</td>
<td>Numerical approximation of a Boundary Value Problem by Ritz-Galerkin discretization with functions of local support</td>
</tr>
<tr>
<td>Emphasized in Part I</td>
<td>Emphasized in Part II</td>
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</table>
FEM in Modeling and Simulation: Physical FEM

Physical system \(\xrightarrow{\text{FEM}}\) Discrete model \(\xrightarrow{\text{SOLUTION}}\) Discrete solution

Ideal Mathematical model

Simulation error = modeling + solution error

VALIDATION

Ideализация и дискретизация

VERIFICATION

solution error

generally irrelevant

CONTINUIFICATION

IFEM Ch 1–Slide 14
FEM in Modeling and Simulation: Mathematical FEM

Mathematical model

Discretization + solution error

Mathematical model

Discrete model

Discrete solution

IDEALIZATION & DISCRETIZATION

IDEALIZATION

REALIZATION

IDEALIZATION

REALIZATION

SOLUTION

VERIFICATION

VERIFICATION solution error

generally irrelevant

Ideal physical system

Discrete model

Discrete solution

FEM

Discretization + solution error
Model Updating in Physical FEM

- Physical system
- Experimental database
- FEM
- Parametrized discrete model
- Discrete solution
- Simulation error
Synergy Between Mathematical and Physical FEM

(Intermediate levels omitted)
Recommended Books for Linear FEM


**Basic level (textbook):** Cook, Malkus & Plesha (1989); this third edition is fairly comprehensive in scope and up to date although the coverage is more superficial than Zienkiewicz & Taylor.

**Intermediate level:** Hughes (1987). It requires substantial mathematical expertise on the part of the reader. Recently reprinted by Dover.

**Mathematically oriented:** Strang & Fix (1973). Most readable mathematical treatment although outdated in several subjects.

**Most fun (if you like British "humor"):** Irons & Ahmad (1980)

**Best value for the $$$:** Przemieniecki (Dover edition 1985, ~$16). Although outdated in many respects (e.g. the word "finite element" does not appear in this reprint of the original 1966 book), it is a valuable reference for programming simple elements.

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