A Complete Axisymmetric FEM Program
Accessing NoteBook Quad4.SOR.nb for HWs

AFEM Chapter 6 Index

*Chapter 6: A Complete Axisymmetric FEM Program. Note: benchmark examples moved to Chapter 14 because of length.

*Chapter 6: Slides in PDF.

*Notebook containing the axisymmetric program QuadSOR documented in this Chapter and used in Chapter 7 for benchmarks. Implements 4-node and 8-node quadrilateral ring elements. Plotting problem under Mma v6.0 and 7.0 fixed. Cells 17 and 18, useful for HW#5 added.

HW Assignment #6 (Chapters 5-7) due Thursday Mar 7. Note: First midterm will be posted by Feb 20 noon, due Th Mar 1 at class time.

Last update: February 18, 2013
Axisymmetric (SOR) Program Configuration

User prepares script for each problem

Problem Driver

Analysis Driver

Assembler

BC Application

Built in Equation Solver

Nodal Stress Recovery

Element Stiffnesses

Element Stresses

Element Library

Utilities: Tabular Printing, Graphics, etc

Presented in Chapter 5
Internally Pressurized Thick Tube: Problem Definition

tube extends indefinitely along the $z$ axis

$2b$

$2a$

$r$

$r$

internal pressure $p$

TUBE CROSS SECTION
Internally Pressurized Thick Tube: FEM Discretization

(a) THICK TUBE UNDER INTERNAL PRESSURE

(b) FEM DISCRETIZATION

"Salami Slice"

$r = a$

$r = (a+b)/2$

$r = b$
ClearAll[Em, ν, a, b, h, p, Kfac, Ner, Nez];
Em = 1000.; ν = 0; Ner = 4; Nez = 1;
Kfac = 1; a = 4; b = 10; h = 1; p = 10.0; aspect = h/(b - a);

(* Define FEM model *)
MeshCorners = N[{{a, 0}, {b, 0}, {b, h}, {a, h}}];
NodeCoordinates = GenerateRingNodeCoordinates[MeshCorners, Ner, Nez];
ElemNodeLists = GenerateQuad4NodeNumbers[Ner, Nez];
numnod = Length[NodeCoordinates]; numele = Length[ElemNodeLists];
ElemTypes = Table["Quad4"], {numele}];
ElemMaterial = Table[{Em, ν}, {numele}];
ElemFabrication = {}; ElemBodyForces = {};
FreedomValues = Table[{0, 0}, {numnod}];
FreedomValues[[1]] = FreedomValues[[2]] = {Kfac*p*a*h/2, 0};
FreedomTags = Table[{0, 1}, {numnod}];
ElemOptions = {True, 2, Kfac};
Model Definition (cont'd)

Plot2DElementsAndNodes[NodeCoordinates, ElemNodeLists, aspect, "Thick tube mesh", True, True];

Thick tube mesh
Invoking the Solution Process

(* Solve problem and print results *)

{NodeDisplacements, NodeForces, NodeStresses} = RingSolution[
  NodeCoordinates, ElemTypes, ElemNodeLists,
  ElemMaterial, ElemFabrication, ElemBodyForces,
  ElemOptions, FreedomTags, FreedomValues];
**Printing Solution Results**

```plaintext
Print["Computed displacements:"];
PrintRingNodeDisplacements[NodeDisplacements];
Print["Node forces including reactions:"];
PrintRingNodeForces[NodeForces];
Print["Averaged nodal stresses:"];
PrintRingNodeStresses[NodeStresses];
```

### Computed displacements:

<table>
<thead>
<tr>
<th>Node</th>
<th>ur</th>
<th>uz</th>
<th>Node</th>
<th>sigrr</th>
<th>sigzz</th>
<th>sigθθ</th>
<th>sigrz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.255711</td>
<td>0</td>
<td>1</td>
<td>12.3626</td>
<td>2.17846</td>
<td>67.5211</td>
<td>-0.781042</td>
</tr>
<tr>
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<td>-0.0244494</td>
<td>2</td>
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<td>67.5211</td>
<td>0.781042</td>
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<tr>
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<td>3</td>
<td>8.04755</td>
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<td>45.402</td>
<td>-0.533743</td>
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<tr>
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<td>-0.0213253</td>
<td>4</td>
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<td>-0.384588</td>
<td>45.402</td>
<td>0.533743</td>
</tr>
<tr>
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<td>5</td>
<td>9.0946</td>
<td>-0.878575</td>
<td>35.9983</td>
<td>-0.405955</td>
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<td>-0.0201795</td>
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<td>-0.878575</td>
<td>35.9983</td>
<td>0.405955</td>
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<tr>
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<td>-0.467814</td>
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<td>0.0637133</td>
<td>29.1534</td>
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<tr>
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<td>2.91302</td>
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<td>23.3714</td>
<td>-0.410162</td>
</tr>
<tr>
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<td>-0.016437</td>
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<td>2.91302</td>
<td>0.336958</td>
<td>23.3714</td>
<td>0.410162</td>
</tr>
</tbody>
</table>
```
Comparing with Exact (Analytical) Solution
(Cell 15B of Quad4.SOR.nb)

urexact[r_] := p*a^2*(1+ν) * (b^2+r^2*(1-2*ν)) / ((b^2-a^2)*E_m*r);
sigrrexact[r_] := p*a^2/ (b^2-a^2) * (1-b^2/r^2);
sigθθexact[r_] := p*a^2/ (b^2-a^2) * (1+b^2/r^2);
urlist = Table[{NodeCoordinates[[2*n-1,1]],
                NodeDisplacements[[2*n-1,1]]}, {n, Ner+1}];
pFEM = ListPlot[urlist, PlotJoined -> True, DisplayFunction -> Identity];
pexact = Plot[urexact[r], {r, a, b}, DisplayFunction -> Identity];
Show[Graphics[AbsoluteThickness[2]], pexact, Graphics[RGBColor[1,0,0]],
      pFEM, DisplayFunction -> $DisplayFunction,
      TextStyle -> {FontFamily -> "Times", FontSize -> 12},
      GridLines -> Automatic, Axes -> True, PlotRange -> All,
      PlotLabel -> "Radial displacements (black=exact, red=FEM)";]

(Similar scripts for radial stress and hoop stress omitted
for brevity)
Comparing with Exact (Analytical) Solution (cont'd)

radial displacement  radial stress  hoop stress

Radial displacement $u_r$ (black=exact, red=FEM)
Radial stress $\sigma_{rr}$ (black=exact, red=FEM)
Radial stress $\sigma_{00}$ (black=exact, red=FEM)
Homework Problem #2

A coarser mesh (9 elements) is solved in Cell 18 of Quad4.SOR.nb

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Homework Problem #3

A coarser mesh (4 elements) is solved in Cell 19 of Quad4.SOR.nb