1. Describe the major research and education activities of the project.

The project focuses on three interrelated Tiers—Student Algebraic Reasoning, Teacher Practice, and Teacher Professional Development. The major research and education activities for each tier are described below.

1.1 Tier 1: Student Algebraic Reasoning

The research at this tier focuses on the development of students’ algebraic reasoning, in particular (1) their developing abilities to solve problems presented in different representational formats (e.g., word problems, equations, graphs) and to translate among and between these representations, and (2) the development of their understanding of core algebra concepts (e.g., equivalence, variable). In addition, we have also begun investigating curricular materials with a particular focus on how curricula may enable or constrain the development of student understanding of core algebra concepts.

In Year 3 of the project, the major research activities in Tier 1 were:

1. The second administration of the longitudinal assessment, a collaborative project across the three university sites. This assessment included items focusing on equality, variables, representational fluency, and problem solving.
2. Collection of a second wave of student interview data. The Year 3 interviews focused on students’ understandings of equality and variable as well as problem solving. Coding and analysis for these interviews are continuing.
3. Analyses of data collected in Year 1, focusing on the relationship between understanding of equality and problem solving, and preparation of a manuscript on this topic.
4. Analyses of middle school curricular materials, with a particular focus on the treatment of equality and variable in both reform and traditional curricula. Our analyses to this point have focused on Connected Mathematics and Saxon Math, grades 6 through 8.

1.2 Tier 2: Teacher Practice

The research at this tier has had two foci: (1) teachers’ use of gestures during instruction, and (2) teachers’ knowledge about students’ algebraic thinking.
Teachers’ use of gesture during instruction. We hypothesize that teachers’ gestures play an important role in their communication about the links between different representations of mathematical information. In addition, teachers also use gestures to “ground” abstract or complex ideas. Because early algebra involves many abstract concepts (e.g., variables) and complex procedures (e.g., isolating variables, factoring), as well as multiple external representational systems (e.g., graphs, tables, symbolic equations), studies of teachers’ gestures may inform teachers’ instructional practices and, ultimately, impact the development of student algebraic reasoning.

In the previous grant year, we began to analyze video of early algebra instruction with a focus on the teacher’s use of spontaneous manual gestures as a component of her instructional communication. This work has continued during Year 3, with a presentation of initial findings at the International Conference of the Learning Sciences.

Teachers’ knowledge about students’ understanding of students’ algebraic reasoning. In Year 3 we also designed and collected data for a project investigating teachers’ knowledge about student thinking in early algebra. The project involved individual interviews with teachers. We presented teachers with sample items and asked them to predict how students might solve the problems, and we presented samples of student work and asked teachers to evaluate students’ thinking, comment on misconceptions, and so forth.

1.3 Tier 3: Teacher Professional Development

Conceptual Framework

The professional development program and research are grounded in a situative perspective on teacher learning. We conceptualize teacher learning as both a process of active individual construction of knowledge and a process of enculturation into the practices of the profession. To understand teacher learning, we must study it within multiple contexts and consider both teachers as individual learners, and the social contexts within which they participate in their own professional growth and development (Putnam & Borko, 2000). Two constructs derived from a situative perspective frame the design and research of our program: teacher learning communities, and knowledge for teaching.

Teacher learning communities. A number of scholars who ascribe to situative theories of learning have identified community as an important ingredient for teacher learning and educational reform (Frykholm, 1998; Grossman, Wineburg, & Woolworth, 2001; Little, 2002; Stein, Silver, & Smith, 1998). Drawing from this work, the STAAR professional development program features the community as a key component. Efforts to create and maintain a teacher learning community characterized by trust and respect, as well as by norms for critical dialogue about teaching, have been central to our work.

Teachers’ mathematical and pedagogical knowledge. The NCTM Principles and Standards document suggests that “teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks” (NCTM, 2000, p. 17). However, more than 25 years of research have indicated that teachers do not typically possess this rich and connected knowledge
of mathematics (Knuth, 2002; Mewborn, 2003; Nathan & Koedinger, 2000), and this lack of mathematical understanding negatively influences instructional practices (Putnam & Borko, 1997). These findings point to the importance of enhancing the quality and depth of teachers’ mathematical and pedagogical knowledge—a focus central to the STAAR professional development program.

1.3.1 Education Activities: Professional Development at University of Colorado

The STAAR Professional Development team at the University of Colorado developed, conducted, and studied a professional development project designed to help teachers foster algebraic thinking in their classrooms during Summer 2003 and Academic Year 2003–2004. The program had two major components: a summer institute that included 60 contact hours of meeting time, and professional development workshops conducted throughout the school year.

*Summer Algebra Institute.* Sixteen teachers participated in a 2-week summer institute—taught collaboratively by two members of the STAAR–CU research team. The institute was offered as a 3-credit course through the Applied Mathematics Department and School of Education. The summer institute was designed to address four major goals:

- Begin to develop a professional learning community.
- Support the development of teachers’ algebraic content knowledge.
- Support the development of teachers’ knowledge about algebra for teaching.
- Provide an opportunity for teachers to learn mathematics in a reform-oriented setting.

The institute was structured around four major types of activities: (1) solving mathematical problems, (2) examining children’s thinking, (3) reading and discussing current literature, and (4) reflecting on one’s own teaching. The teachers worked collaboratively with their colleagues throughout the institute, as they addressed a wide range of algebra problems. They frequently focused on a single problem for 30 to 60 minutes, first working in small groups and then sharing their solution strategies with the whole class. Several activities focused explicitly on examinations of student thinking, including discussions of videos of students engaged in problem-solving tasks (videos were provided by members of the STAAR–UW team).

*School year professional development.* Eight of the teachers participated in seven professional development workshops during the 2003–2004 academic year (three in the fall and four in the spring). These workshops were taught collaboratively by two members of the STAAR–CU research team (one of the summer institute leaders and one graduate student new to the team in Fall 2003). They addressed several goals, including the following:

- Supporting the development of teachers’ pedagogical content knowledge through selection and enactment of problem solving tasks.
- Fostering mathematical discourse.
- Understanding student thinking.
- Encouraging teachers to identify and pursue their own professional learning goals.
In addition to the workshops, members of the research team visited the classrooms of participating teachers approximately nine times during the school year, to observe and videotape their teaching. Selections from these videotapes were used by both the teachers and the researchers as key components of several professional development workshop activities.

1.3.2 Professional Development Research at University of Colorado

Research Goals
Two sets of goals guide our research agenda. The first set of goals was developed in parallel with the goals for the professional development program. These goals include an investigation of the following:

- Development of community and norms of discourse among the cohort of teachers.
- Content and pedagogical content knowledge of teachers in the program.
- Degree to which teachers are incorporating ideas, experiences, and knowledge gained in the professional development setting into their own teaching practices at the middle school level.

The second set of goals is more broadly construed and addresses the degree to which our program may be adopted and adapted in contexts that differ from our own. Specifically, these goals include an investigation of revisions of our model that incorporate different algebraic content, an expanding group of teachers, and multiple professional development facilitators. It is from this line of research that we hope to be able to make recommendations in our final year of the project regarding ways in which this model of professional development might be effectively scaled across contexts and purposes.

Research Methods
Data collection. Primary data sources are video recordings of professional development sessions and classroom lessons, and interviews with professional development facilitators and teachers. We have collected and cataloged over 400 hours of video, including:

- Professional development—video of all sessions of the summer institute and ongoing professional development workshops, using two to four cameras at all times.
- Classroom instruction—video of approximately nine class sessions taught by each of the participating teachers, using two cameras at all times.

We have also conducted extensive interviews with the professional development facilitators and participating teachers including:

- Interviews with professional development facilitators following each professional development session.
- Interviews with participating teachers at the end of the summer institute and after the final professional development workshop.
- Interviews with participating teachers following each classroom observation.

In addition to these data sources, we have collected the following artifacts of practice:

- Artifacts of the professional development
  - Teachers’ reflections and autobiographical statements.
Data analysis. We are using a variety of methods to categorize and analyze the large, varied corpus of data. These methods include:

- Catalog and description of summer institute activities (completed).
- Catalog and description of professional development workshop activities (completed).
- Coding and analysis of selected video segments from summer institute and professional development workshops (ongoing).
- Transcription, coding and analysis of interviews with professional development facilitators and teachers (ongoing).
- Vignette analysis of critical incidents during the summer institute and professional development workshops (ongoing).
- Analysis of pre- and posttests of algebra content knowledge assessments (completed).

Our future plans for analysis include creating a catalog of observations of classroom teaching and coding and analyzing selected video segments of instructional practices.

1.3.3 Education Activities: Professional Development at University of Wisconsin

Much of our effort in year three involved planning and implementing a new program of teacher professional development. Twenty middle school teachers are currently enrolled in a year-long graduate credit course, “Understanding and Cultivating Students’ Algebraic Thinking.” Some teachers are returning from our Year 2 TPD work at UW, and some teachers are new to the community.

Our objectives for teachers in this course include both content knowledge and professional process objectives.

- Content Outcomes—Teachers will
  - Generate multiple representations.
  - Demonstrate representational fluency.
  - Generalize solution strategies.
  - Explain algebraic reasoning.

- Process Outcomes—Teachers will
  - Suspend their current teacher persona (so they are able to adopt a new “learner” persona).
  - Actively participate in the learning community.
  - Demonstrate sincere concern for transitioning students from arithmetic to algebraic reasoning.
  - Practice professional norms.
The UW course design incorporates five additional concepts:

- Facilitated learning community approach (Wilsman et al., 2002).
- Practice-based approach (Smith, 2001).
- Case-based learning with contrasting-cases method (Beitzel & Derry, 2004; Catrambone & Holyoak, 1989; Gentner et al. 2003; Schwartz & Bransford, 1998).
- Based in cognitive science concepts, such as learning and transfer, expert-novice differences (Bransford et al., 2000).
- Use of metaphors to build awareness of the course outcomes (http://www.wcer.wisc.edu/stellar/algebra/overview.htm).

The course began with a four-day summer workshop and is continuing through the school year. It incorporates a hybrid implementation of monthly face-to-face meetings that continue through online interactions.

1.3.4 Professional Development Research at University of Wisconsin

Course Evaluation Research Design

The overall research design is a pretest and posttest design with a matched comparison group. One test in our assessment battery is a video assessment of student work. Teachers are asked to analyze videos of two student interviews collected by UW STAAR researchers. We have developed a rubric for scoring teachers’ analyses that evaluates the teachers on four aspects: (1) ability to discuss student representations; (2) ability to judge student developmental trajectories; (3) ability to select appropriate pedagogical moves; and (4) ability to metacognitively reflect on their own knowledge. We are currently validating this rubric.

The second test is an assessment of mathematics content knowledge. It is short version of an instrument developed by Goldsmith, Driscoll, and Seago for use in their NSF ROLE Project (Education Development Center, 2003).

The group enrolled in our pilot teacher professional development program (N = 20) was administered the assessments in Summer 2004 and will be tested again in May 2005. They also completed the video assessment of student work before and after the summer workshop. A matched comparison group of Madison teachers (N = 20) will take the same tests in May 2005. In May, both treatment and comparison groups will also complete questionnaires describing and reporting their satisfaction with their professional development experiences.

Research on TPD Activity Design: Scaffolding Contrasting-Cases Instruction

The summer workshop attempted to promote teacher learning through analysis and discussion of contrasting cases of teacher problem solving, student problem solving, and teachers’ pedagogical practices. Cognitive research indicates that contrasting cases is powerful for promoting idea abstraction, contextualization, and transfer. We are investigating how to design and scaffold contrasting-case discussions for developing teachers’ mathematical and pedagogical knowledge.
During the summer workshop, we designed and formatively evaluated contrasting cases activities (called SAMs – Sample Algebra Modules) *in situ* as the workshop progressed. Our workshop revolved around three SAMs, each addressing a different foundation concept in the NCTM 2001 and 2004 descriptions of algebraic reasoning.

SAMs were also designed to allow evaluation of the impact of contrasting cases on teacher learning. The basic steps in a SAM were:
1. Teachers solved an assigned problem.
2. Discussion of an article from the NCTM middle school journal (*Teaching Mathematics at the Middle Level*) expected to enhance teachers’ ability to analyze their problem solutions and generate alternative solutions.
3. A “pre-assessment” in which teachers applied the reading to reflect on their problem solving.
4. Small-group analysis and contrasting of solutions found within small groups.
5. Whole-group analysis and contrasting of solutions contributed by small groups.
6. A “post-assessment” opportunity for teachers to re-reflect on their original solutions and generate alternatives.
7. Whole-group discussion of the SAM activity.

We collected participants’ (facilitators’ and teacher-learners’) reactions to SAM designs and used those to adjust the design and scaffolding of each subsequent SAM. We also collected pre- and posttest data within each SAM activity, to help us assess the learning effects of the contrasting-cases components.

2. Describe the major findings resulting from these activities.

In this section we describe major findings from Year 3 from: (1) analyses of the relationship between understanding of the equal sign and algebra equation solving, (2) comparisons of reform-based and traditional middle school curricular materials, and (3) preliminary results from UW-Madison and CU-Boulder TPD research.

2.1 Relationship Between Understanding of the Equal Sign and Algebra Problem Solving

During Year 3, we prepared a manuscript, currently under review, about the relationship between understanding of the equal sign and performance solving algebraic equations. The study was based on data from 177 middle school students who completed an assessment during Year 1 of the grant period.

In one of the assessment items, students were asked to define the equal sign. Their definitions were coded as reflecting a relational understanding of the equal sign (e.g., “it means two amounts are the same”) or not (e.g., “it means the total of the problem”). In another assessment item, students solved an algebraic equation (e.g., $4m + 10 = 70$). Our analysis focused on two outcome measures: (1) whether or not students solved the equations correctly, and (2) whether or not students used an algebraic strategy to solve the equation (as opposed to an informal, nonalgebraic strategy such as guess and test). Based on prior work (e.g., Kieran, 1981), we predicted that students who lack a relational understanding of the equal sign might have difficulty understanding the steps involved in an algebraic strategy (e.g., why do the same thing to both sides?). Consequently, we
expected that such students might tend to utilize nonalgebraic strategies to solve the equations.

At each grade level (6–8), more students who provided relational definitions of the equal sign solved the equations correctly. We analyzed the data using logistic regression (for categorical outcome measures) and found this effect to be significant, $Wald(1, N = 177) = 22.64, p < .001$.

It might be argued that the relationship between equal sign understanding and equation-solving performance could be due to students’ general abilities in mathematics, and not due to an inherent relationship between equal sign understanding and equation solving, per se. To address this issue, we performed a similar analysis on a subset of the students for whom we have standardized test scores ($N = 65$), so that we could control for mathematics ability. In addition to grade level (6, 7, or 8) and equal sign understanding (relational or not), we included national percentiles on the mathematics, reading, and language components of a national standardized test as predictors in the model. Even when controlling for grade level and standardized mathematics test scores, the association between equal sign understanding and equation-solving performance remained significant, $Wald(1, N = 65) = 3.85, p = .05$. Thus, the observed relationship between equal sign understanding and equation-solving performance was not simply due to better students performing well on both items.

None of the sixth-grade students and only 1% of seventh-grade students used an algebraic strategy to solve the equations, so the relationship between equal sign understanding and use of an algebraic strategy could not be tested among sixth- and seventh-grade students. For the eighth-grade students, however, there was a positive relationship between equal sign understanding and use of an algebraic strategy, $\chi^2 (1, N = 58) = 18.45, p < .001$.

We could not analyze the effects of mathematics ability on use of an algebraic strategy because there were too few eighth-grade students for whom we had standardized test scores. However, some eighth-grade students were enrolled in an algebra course, and others were not. It seems probable that students enrolled in algebra would be more likely both to use an algebraic strategy and to provide a relational definition of the equal sign. If this were the case, the observed relationship between equal sign understanding and use of an algebraic strategy might be due to both being related to algebra course work, rather than due to an inherent relationship between equal sign understanding and use of an algebraic strategy, per se.

As expected, eighth-grade students enrolled in algebra ($N = 7$) were more likely than those not enrolled in algebra ($N = 45$) to use an algebraic strategy (86% versus 29% of students). In addition, students enrolled in algebra were more likely than students not enrolled in algebra to give a relational definition of the equal sign (71% versus 29% of students). However, the relationship between equal sign understanding and use of an algebraic strategy remained significant even when students enrolled in algebra were excluded from the analysis, $\chi^2 (1, N = 45) = 9.49, p = .002$. Thus, the observed relationship between equal sign understanding and use of an algebraic strategy was not due to students enrolled in algebra performing well on both items. These data suggest that equal sign understanding informs students’ use of an algebraic strategy.
Thus, we found a strong positive relationship between middle school students’ equal sign understanding and equation-solving performance, and we showed that this relationship holds even when controlling for mathematics ability (as assessed via a standardized test). In addition, we found a strong positive relationship between equal sign understanding and use of an algebraic strategy by eighth-grade students (students who have had more experience with algebraic ideas and symbols as compared to their peers in sixth and seventh grades), and we showed that this relationship holds for those eighth-grade students who are not enrolled in an algebra course. Taken together, these findings suggest that understanding of the equal sign is a pivotal aspect of success in solving algebraic equations. Accordingly, these findings help build a case for the importance of continuing to explicitly develop students’ understanding of the equal sign during their middle school mathematics education.

2.2 Curriculum Analyses: Presentation of the Equal Sign in Curricular Materials

In our curriculum analysis work, we are exploring how sixth- through eighth-grade mathematics textbooks present the equal sign. Our ultimate goal is to evaluate and compare several different mathematics curricula with respect to this issue. To date, we have focused on the sixth- though eighth-grade textbooks in the *Connected Mathematics Program* and the *Saxon Math* program.

We coded a random sample of 50% of the pages in each text. Every instance of the equal sign was coded for whether it was used in an *operation-equals-answer* context (e.g., $3 + 4 = \_\_\_$), an *operations-on-both-sides* context (e.g., $3 + 4 = 5 + 2$), or some other context (e.g., $\_\_\_ = 3 + 4$). We defined the *operation-equals-answer* context as any equation containing operations on the left-hand side of the equal sign, and either one number (e.g., $3 + 4 = 7$) or an unknown quantity to solve for (e.g., $3 + 4 = \_\_\_, 3 + 4 = x$) on the right-hand side of the equal sign. We defined the *operations-on-both-sides* context as any equation containing operations on both sides of the equal sign. We defined the other context as any equation that does not fall into one of the other two categories, including (1) equations without numbers (e.g., side $AB = \text{side } CD$), (2) conversion equations (e.g., 12 inches = 1 foot), (3) equations with single numbers on both the left- and right-hand sides of the equal sign (e.g., $7 = 7$), and (4) equations with a single number on the left-hand side of the equal sign and an operation on the right-hand side of the equal sign (e.g., $7 = 3 + 4$).

We hypothesize that exposure to equations in the *operations-equal-answer* context might foster an operational view of the equal sign (e.g., “the equal sign means the total”), and therefore might make it difficult for students to construct a relational view of the equal sign, which is necessary for success at algebra.

*Saxon Math* presented a greater proportion of equal signs in the *operations-equal-answer* context than *Connected Math*, $Wald(1, N = 2905) = 387.76, p < .001$. Indeed, over 60% of the equal signs in the *Saxon Math* texts were presented in this format. In both series, the proportion of equal signs presented in the *operations-equal-answer* context decreased from sixth to eighth grade, $Wald(2, N = 2905) = 76.65, p < .001$. 
The operations-on-both-sides context was of particular interest given past work suggesting that students exposed to such equations activate a relational view of the equal sign (McNeil & Alibali, in press). Surprisingly, neither series included many equal signs in the operations-on-both-sides context (always fewer the 8% of all equations). However, the proportion of equal signs in the operations-on-both-sides context was greatest in eighth grade, *Wald*(2, *N* = 2905) = 45.02, *p* < .001.

The preponderance of the operations-equal-answer context in curricular materials may promote an operational view of the equal sign, and this may cause problems for students as they make the transition to algebra. Future studies must be designed to directly test this relation.

### 2.3 Research Findings: STAAR–CU Professional Development Program

Our analyses to date have focused on the two central goals of the summer institute, and on data collected during that institute. Specifically, we have explored the ways in which the instructors created a professional learning community with the teachers and how this community contributed to the development of participants’ knowledge of algebra.

**Creation of a Professional Learning Community**

The instructors in the STAAR summer institute worked carefully and systematically to create a professional learning community with the teachers. Our analysis of their efforts focuses on four strategies that are fundamental to establishing and maintaining a successful learning community: posing rich tasks, creating a safe environment, asking students to explain and justify solutions, and actively processing the ideas of others (Cobb, Boufi, McClain, & Whitenack, 1997; Silver & Smith, 1997). These strategies are as relevant to learning communities for teachers as they are to K-12 mathematics classrooms (Sherin, 2002). All four strategies were evident in the ways that the instructors structured activities throughout the institute. They were especially prominent during the initial activities, when creating discourse norms and establishing trust were central to the instructors’ goals and intentions. For example, during the first problem-solving activity of the institute, the instructors selected a rich task with multiple solution strategies, created a safe environment within which teachers could explore this task, and called upon the teachers to share their solutions and solution strategies and to process and build upon one another’s ideas.

**Teacher Learning: Algebra Content Knowledge and Instructional Practices**

Analyses of pre- and post-institute algebra content tests and interviews, teachers’ daily reflections, and their final papers provide initial evidence that the summer institute had a powerful impact on participating teachers. We expect that ongoing analysis of observations of the teachers’ classes and of interviews about their beliefs and instructional practices conducted throughout the ensuing school year will provide additional support for our assertions, detailed below, based on these data sources.

*The algebra content knowledge assessment.* All 16 teachers were given an assessment of content knowledge on the first and last days of the institute. The assessment consisted of 27 contextually based problems designed to evaluate their understanding of several foundational topics in algebra including variable, equality, pattern recognition, representational fluency, and systems of equations. There was a modest difference in the scores on these identical tests between the pretest (average score: 21.25), and posttest
A second analysis of the content knowledge assessment reflected the institute's emphasis on using multiple methods and representations to solve problems. On the pretest, only one problem elicited multiple strategies from the teachers. On the posttest, however, teachers presented multiple solution strategies on nine problems. A scoring rubric to calculate the number of strategies a teacher used to solve a given problem revealed 350 strategies on the pretest (an average of 21.9 strategies per teacher) and 483.5 strategies on the posttest (an average of 30.2 strategies per teacher), thus indicating the teachers' growing ability to think of problem solutions in multiple ways.

**Teachers' self-reports about summer institute impact.** The three self-report data sources (reflections, final course papers, interviews) revealed teachers' impressions about the institute's impact on their content knowledge, mathematics-specific pedagogical knowledge, and recognition of the importance of community. The teachers' self-reports provide additional evidence that the summer institute had a positive impact on their knowledge of algebra for teaching. Most teachers commented about their new understanding of specific algebraic topics. Some reported that they learned new techniques for solving particular types of mathematics problems, and that they noticed mathematical connections of which they had previously been unaware. In addition, several teachers reflected on the value of specific instructional strategies, representations, and curricular materials that they experienced as students in the institute, and on their intentions to use these tools with their own students. Many teachers also commented that the strong community within the summer institute facilitated their learning and gave them skills to establish similar communities within their own classrooms.

**2.4 Research Findings: STAAR–UW Professional Development Program**

We are currently validating our scoring rubrics and have not completed any analyses. Only a few preliminary results are available at this time. These include teacher ratings and comments, indicating that the summer workshop was considered a successful learning experience. In addition, through formative assessments we have acquired knowledge about how to design workshops based on contrasting case activities. For example, we have learned that small-group contrasting-cases discussions are more productive if scaffolded through written activity guides that require teachers to make explicit and detailed comparisons of their solutions. Also, whole-group solution discussions greatly enhance the perceived effectiveness of the class. However, because some participating teachers feel insecure in publicly discussing mathematics, building trust and community is a prerequisite to productive whole-group contrasting of solutions.

**3. Describe the opportunities for training and development provided by your project.**

The students and staff working on the STAAR-UW project have developed skills for working with human participants both children and teachers; skills for designing items to assess student thinking; and skills for gathering, managing, coding and analyzing data. In addition to the research activities, students and staff have been involved in the design, development, and instruction of a concurrent professional development program—a program that draws upon the results of the research program.
Within the STAAR-CU project, the postdoctoral research associates and graduate research assistants have been involved in all aspects of the professional development program and associated research, under the leadership and guidance of the principal investigators. Graduate research assistants at CU and UW have taken leadership roles in planning and conducting the summer institute and professional development workshops. The two postdoctoral research associates at the University of Colorado are lead authors on two of the articles we have completed. A large number of conference presentations and publications have been authored or co-authored by graduate students on both campuses.

4. Describe outreach activities your project has undertaken.

The summer institutes and professional development workshops in Boulder, CO and Madison, WI are outreach activities to teachers in local school districts. With other NSF-funded projects, this grant co-sponsors graduate courses and continuing online professional development support for middle school mathematics and special education teachers. Our work is generating artifacts and instructional materials and programs that will continue beyond the life of the grant and benefit future cohorts of pre- and in-service teachers.

5. Products

5.1 Articles and Chapters


### 5.2 Presentations and Talks


5.3 Accepted


5.4 Under Review


5.5 Web Sites

http://algebra.colorado.edu/
http://www.wcer.wisc.edu/stellar/algebra/overview.htm
http://labweb.education.wisc.edu/knuth/taar/
6. Contributions of the Work

6.1 The Principal Disciplines (Mathematics Education)
In response to growing concern about students’ inadequate understandings and preparation in algebra, and in recognition of the role algebra plays as a gatekeeper, recent reform efforts in mathematics education have made focal points of algebra curricula and instruction. There is a growing consensus that algebra reform, however, requires a reconceptualization of the nature of algebra and algebraic reasoning as well as a reexamination of when children are capable of reasoning algebraically and when ideas that require algebraic reasoning should be introduced into the curriculum. Recent research has begun to investigate algebra reform in the context of elementary school mathematics, focusing in particular on the development of algebraic reasoning; yet, to date, little research has focused on the development of algebraic reasoning in the middle grades—the focus of our work. Thus, the goals of our work—to develop detailed accounts of learning and instruction in classroom contexts that will guide the design and evaluation of instructional approaches and professional development programs aimed at facilitating the development of algebraic reasoning—hold promise to make a significant contribution to the field of mathematics education.

This project is contributing to theory on teacher learning, as well as to the mathematics education community’s understanding of key strategies for facilitating the development of teacher learning communities and enhancing teachers’ knowledge and instructional practices.

6.2 Other Disciplines
We believe that the approach to professional development we are designing can be adapted to other subject areas such as science.

6.3 The Development of Human Resources
We are contributing to the preparation of future educational researchers through the experiences we are providing to postgraduate research associates and graduate research assistants. In addition, the teachers who participated in the summer institute and professional development workshops are receiving training that will enable them to be instructional leaders in their schools and districts.

6.4 Infrastructure for Research and Education
Our plans for the final years of the project include developing resources based on our experiences and findings, to be disseminated to others so that they can provide similar professional development opportunities for teachers.

7. References


Gentner, D., Loewenstein, J., & Thompson, L. (2003). Learning and transfer: A general role for analogical encoding. *Journal of Educational Psychology, 95*, 393-408.


