ACTIVITIES

Tier 1: Student Algebraic Reasoning

The major research and education activities for the study of student cognition included the following:

1. Coding and data entry of Year 1 data for the student written assessment ($N = 1313$) on representational fluency in the Denver Public Schools (DPS).
2. Administration of representational fluency assessment for Year 2 data.
3. Collection of student interview data ($N = 5$) on representational fluency.

Tier 2: Teacher Practice (Experimental Studies: Expert Blind Spot; Gestures)

As part of the inquiry into how teacher knowledge affects instruction, an experimental study on the effects of content knowledge on decision making and instructional practice was completed with pre-service teachers in secondary-level mathematics and science. This work fits into the emerging *expert blind spot* (EBS) paradigm.

In addition, researchers at the University of Colorado–Boulder (UCB) and the University of Wisconsin–Madison (UWM) collaborated to analyze video of early algebra instruction with a focus on the teacher’s use of spontaneous manual gestures as a component of instructional communication. Detailed findings are included in the UWM report. Several reports have been generated from this work.

Tier 3: Teacher Professional Development (Studies of Teacher Beliefs)

1. Completed study of the initial and changing views about students’ algebraic development held by five DPS high school algebra teachers.
2. The intervention served as a framework for the design and implementation of a Web-based model for exploring confrontation of specific views of learning and teaching that can inform a larger program of teacher professional development that will soon be field tested (for more details, see report from Carnegie Mellon).

Tier 2 / Tier 3: Summer 2003 Algebra Course

During Year 2, Tier 2 and Tier 3 from the UCB team collaborated to present a summer algebra course geared for middle school algebra teachers. Offered through the Continuing Education program in the School of Education at UCB, the two-week, three-credit, graduate-level course, entitled “Facing the Unknown: Algebra,” was held in July 2003.
Planning. The general scope of the summer algebra course was developed jointly by members of the STAAR Tier 2 and Tier 3 teams. Built on the work of Tiers 0, 1, and 2, the course was grounded in emerging theories about how students develop algebraic reasoning. Two years of research by Tier 2 into the beliefs and practices of middle school mathematics teachers generated a team consensus that middle school teachers would benefit from extended learning opportunities centered on teaching algebraic reasoning, with particular focus on development of abstract thinking and conceptual understanding.

Goals. The course focused on exploring foundational algebraic content in depth. Specific course goals included the following:
1. Increasing teachers’ algebraic content knowledge, reasoning abilities, and critical thinking skills.
2. Creating a teacher community or network.
3. Having teachers experience learning in a classroom based on “reform” ideals.
4. Increasing teachers’ awareness of students’ algebraic thinking by examining student work, discussing student thinking, and reading current literature.
5. Influencing teachers’ beliefs about algebra and pedagogy.

Participants. Sixteen mathematics teachers participated in the course. All were in-service teachers from a cross section of Colorado school districts (including urban, suburban, and rural districts), teaching mainly at the middle school level. Although there was a broad range of experience among the teachers, the majority had relatively little experience teaching middle school algebra. Curricula and textbooks used in their classrooms ranged from traditional to reform based.

Activities. The course was divided into eight topic areas. The first week focused on patterns and functions and included multiple representations such as graphs and tables in both areas. The second week focused on equality, variables (an overarching topic), systems of equations, exponents, and integers. Within each topic area, students spent considerable time working on mathematical tasks, solving mathematical problems, and engaging in mathematical thinking.

Throughout the course, teachers accessed the STELLAR Web site, which housed journal articles and other relevant documents, along with a discussion board. Students were required to read selected articles and participate in threaded discussions. This online component was intended to reinforce the community teachers created in the classroom and provide an opportunity to access that community outside the classroom.

Teachers were required to keep a notebook documenting their work during the course. This work included their solution strategies for each problem, class notes (including other students’ solution strategies, new mathematical terminology, etc.), and daily reflections consisting of short summaries and insights regarding the day’s learning and experiences. These notebooks and reflections serve as research data, along with longer papers written by teachers at the beginning and end of the course, pre- and post-course interviews conducted individually with each teacher, and pre- and post-course mathematics assessments. In addition, the entire two-week course was videotaped by two cameras, one focusing on the instructor, another on classroom activities. Extensive daily notes were taken by at least four researchers with the aid of a computer application.
developed by the team to synchronize comments with camera frames and to sort and classify research notes.

**Tier 2 / Tier 3: Fall 2003 Professional Development Workshops**

Following the summer algebra course, the UCB team planned and began carrying out monthly professional development (PD) workshops that will continue through Year 3 of STAAR. Seven workshops are planned for the 2003-2004 academic year.

**Planning.** Similar to the planning of the summer algebra course, the monthly PD workshops are being developed collaboratively by Tier 2 and Tier 3. Unlike the summer course, in which increasing content knowledge was the overriding goal, these workshops focus on providing teachers with skills and strategies to improve their pedagogy. Because the participants in the PD workshops also took part in the summer algebra course, the workshops provide a natural opportunity for them to explore ways to incorporate their newly acquired knowledge and perspectives on algebraic reasoning into their teaching.

**Goals.** The primary goal of the PD workshops is to provide teachers with opportunities to reflect on, critique, and reconsider their pedagogy, specifically in the context of algebra teaching. Teachers are encouraged to take on the role of teacher-researchers, to accept a shared responsibility for improving their classroom instruction, and to put into practice new ideas and skills they learn related to algebra and algebra pedagogy. Additional goals include creating and sustaining a teacher community or network, influencing teachers’ beliefs about algebra and pedagogy, and helping teachers gain familiarity with specific recommendations from the National Council of Teachers of Mathematics (NCTM, 2000).

**Participants.** The ten participants in the PD workshops also attended the summer algebra course. The remaining six teachers from the summer course did not participate for various personal and professional reasons.

**Activities.** A central activity of the PD workshops involves having teachers watch excerpts from videotapes of middle school mathematics lessons, including their own, while carefully considering relevant pedagogical issues. They also read related articles, including portions of NCTM’s *Principles and Standards for School Mathematics* (2000), and discuss these articles in the context of their own teaching.

Throughout the workshops teachers generate their own pedagogical goals. With the assistance of their peers, the teachers revisit, reconsider, and refine their goals during each workshop. When possible, teachers watch and share videotaped examples of their goals enacted in particular lessons.

**Additional Tier 2 Activities**

In addition to the collaborations with Tier 3 as detailed above, Tier 2 completed data collection for two studies framed within the guiding scope of work for Tier 2 as outlined in the original proposal. Serving as the foundation upon which much of the Tier 2/Tier 3 Algebra course described above was based, the elaboration of teachers’ “turning points” in PD was explored.
Additional Tier 3 Activities

*Continued research into the field of hypermedia cases:* Hypermedia, case-based learning offers a promising opportunity in the world of PD. As a precursor to a formal study focused on the development of hypermedia cases and their use, we conducted a review of the literature on hypermedia, case-based learning. This research traces the academic or intellectual terrain (e.g., new views on the complexity of teaching, case-based learning, and situated learning) that has laid the groundwork for the emergence of hypermedia, case-based PD. Using Cochran-Smith and Lytle’s (1999) three categorical representations of the knowledge/practice relationship in PD programs, we investigated numerous hypermedia, case-based PD programs in an effort to understand similarities and differences among these programs, and their impact on teacher learning.

*Development of a conceptual framework:* In order to provide a clear vision of possible ways to work with the STELLAR Web-based, PD environment utilizing hypermedia cases, we developed a conceptual framework that reflects our situative perspective on teacher learning—emphasizing that knowledge is socially constructed and distributed across communities of learners, and is situated in the context in which it is constructed; and that learning is influenced by prior knowledge and beliefs. Using this conceptual framework, we explore the use of the Internet as a learning platform for teachers, examine factors influencing the success or failure of large-scale change efforts, and consider the importance of structuring teachers’ learning activities around clearly defined learning goals that are consistent with state or district standards and with teachers’ own PD goals. Tier 3 researchers gave a presentation on this work at the AERA 2003 annual meeting and intend to complete a publishable paper describing this conceptual framework by summer 2004.

**FINDINGS**

**Tier 1: Student Interview Data: Representational Fluency**

*Method.* Five students in grades six through eight were interviewed (with audio recording and observation notes) while they solved problems from the assessment prepared by the research team, along with seven practice problems. The practice problems revealed that these students struggled with basic arithmetic computations and with equation-based and graph-based procedures. All students were able to attend to the requested tasks. The researcher had to be ready to take notes immediately in order to record the actions and additional activities students undertook that were beyond their verbal reports. Because there was much in the way of pointing and explaining in response to the interviews, one major change that could be considered for this research would be to use videotape in conjunction with audio.

*Results.* Overall it is clear that these students have fragments of mathematical knowledge but little understanding of the unifying rules about how to work with numbers and formal representations in a robust manner. Four students from the same school had a common attitude: They did not worry about whether they found the correct answer, and they had a sense of confidence about trying problems. This confidence may be due to the
volunteer selection process. The willing students were probably the more confident students.

Students used a combination of guessing, estimating, and occasionally applying fragments of knowledge that they thought might relate to the given problem. Sometimes this system produced correct answers, but most often it resulted in errors. Often the students appeared to be seeking patterns or relationships from their estimates and arithmetic computations, but they were unable to develop usable or accurate patterns that they could generalize from. Sometimes this inability was due to computational errors that obscured the patterns, but more often it came from use of an invalid strategy or misinterpretation of the information given in the problem. At a more basic level, it was expected that middle school students would all be capable of using appropriate arithmetic computations at a mastery level. However, in the strategies shown on these examples, students chose to use successive addition steps in situations where multiplying would have been more efficient and appropriate. In such a small sample of students a conclusion cannot be made, but it raises the question about the level of arithmetic skill levels that can be expected in middle school.

Students were most comfortable with tables of values, the simplest of the representational formats used in the interview (and on the assessment). In contrast, all students struggled with the graphical input. The Colorado Model Content Standards for Mathematics suggests that by sixth grade, students will be able to draw graphs, find pairs of points and set up axes, read scales, and write a scenario from a given graph. None of these students seemed readily able to do any of the graph problems, except for reading the most straightforward values. None of the students was able to correctly interpret the y-intercept as the initial cost to begin the problem. Even though the problem set started with the chart showing the $2 charge for 0 copies, the students did not use this information in their solutions. It would be a valuable research question to study whether students tend to ignore information they don't understand and assume instead that they can complete the problem without it.

All students seemed more comfortable with input from equations than with input from graphs. However, in problem 2 on the warm-up sheet, none of the students recognized the relationship between the two equivalent (but not visually identical) equations, nor did they compare the two equations to obtain an answer to the question "Is the number … the same?" Even those who got a correct answer did so by working through each equation independently and finding the same result, rather than by using structural relations between the equations to recognize equivalence. When asked to write an equation, they were not able to do so, but instead described in words how they worked the problem. This finding is consistent with earlier research on verbal-precedence behavior (Koedinger, Alibali & Nathan, 1999; Koedinger & Nathan, in press; Nathan & Koedinger, 2000a, 2000b; Nathan, Long & Alibali, 2002; Nathan et al., 2002).

Only one student demonstrated a systematic way of evaluating the success of her strategies. The others generated answers to the first step of a problem and used this answer even if it was incorrect or inconsistent with the graph provided. They then used this self-generated value while ignoring the given inputs for subsequent parts of the problem, producing a series of errors and remaining unaware of the inconsistencies.
Tier 2: Teachers’ Use of Gestures as a Form of Scaffolding

The detailed findings from this cross-campus collaborative are included in the UW report. Several reports have been generated from this work.

Tier 2: Role of Subject Matter Knowledge on Teachers’ Views of Mathematics Learning and Development

This study examined the relationship between pre-service teachers’ (PSTs) subject-matter expertise in mathematics and their judgments about students’ algebra problem-solving difficulty. The motivating research question was whether educators with a higher level of mathematics education were more likely to exhibit a misconception about student algebra learning than were educators with less advanced mathematics education. In past research, it was found that teachers with more advanced training in the field of mathematics were more likely to hold a symbol precedence view (SPV) of student algebra development than shown in the student performance data. This study was intended to provide a better test of the hypothesis than was addressed in an earlier study of K-12 teacher beliefs (Nathan & Koedinger, 2000c) where it was not possible to implement certain controls.

All participants ($N = 48$) were pre-service teachers (PSTs) enrolled in a nationally acclaimed teacher education program at a Carnegie Research 1 University. Subject matter knowledge (SMK) in mathematics was rated high if the teachers had completed calculus or above, and low if they had not completed pre-calculus. These participants can be thought of as “developing experts” in mathematics. Of the 35 participating PSTs with advanced mathematics knowledge, 16 were in a specialized program for mathematics and science majors (the MathSci condition in this study) seeking secondary licensure in mathematics or science education. The remaining 19 PSTs with advanced mathematics knowledge (the HiMathK group) were from the general population of teacher education students and were seeking licensure at the elementary grade levels, as were a group of 13 PSTs with basic mathematics knowledge (the BasicMath group).

As predicted by the expert blind spot hypothesis, participants with more advanced mathematics education, regardless of their program affiliation or teaching plans, were more likely to view symbolic reasoning and mastery of equations as a necessary prerequisite for word equations and story problem solving. This expectation is in contrast to students’ actual performance patterns. The average ranking across all of the mathematics/science-major PSTs was virtually indistinguishable from the theoretical ranking predicted by SPV, $r = .94$, $p < .005$. Analyses of individual rankings of each PST showed an average correlation with SPV of 0.72, $SE = .18$. To correct the distribution, a Fisher transformation was applied to each participant’s rank correlation. Mean transformed scores had a 95% confidence interval that included 1.0 ($1.19 <= X <= .64$), making this statistically indistinguishable from a perfect correlation with the SPV ranking. PSTs who were not pursuing certification as secondary mathematics or science teachers but reported advanced mathematics education (HiMathK; $n = 19$) also exhibited an average ranking which strongly correlated with SPV, $r = .94$, $p < .005$. The average Pearson correlation was 0.81. Mean Fisher transformed correlations were indistinguishable from $r = 1.0$. 
PSTs with relatively limited mathematics education (BasicMath; \( n = 13 \)) showed an average ranking that correlated only moderately with the SPV ranking, \( r = .48 \). The average individual Pearson correlation was 0.48. Individually Fisher-transformed ranking correlations with SPV for BasicMath PSTs produced a 95% confidence interval that did not include 1.0 (0.71 \( \leq X \leq .35 \)), indicating this ranking was different from the idealized SPV ranking and that of the PSTs with greater mathematics education.

In addition to providing comments and data on difficulty-ranking, participants responded to a 47-item beliefs instrument on a number of topics related to mathematics instruction and learning. The rating data obtained from the survey showed that the six hypothesized constructs were well formed (Cronbach’s alpha between .54 and .89, with 6 items removed on the basis of reliability analyses). Thus, when we consider participants’ levels of agreement, we have high certainty that agreement with specific items is reflective of each construct in general. Many of participants’ views were in line with current mathematics reform. However, there were also significant differences between participants at the 5% level of certainty that parallel divisions in their level of mathematics education.

A greater percentage of HiMathK and MathSci PSTs believed using algebraic formalisms is best for solving complex problems (an average of 51% agreed) compared to BasicMath PSTs (38%). BasicMath participants were more likely to agree (82%) that learning is fostered through discovery than were HiMathK (64%) and MathSci (56%) PSTs. BasicMath PSTs (66%) were also more inclined to believe that instruction should build on students’ intuitions and invented methods than were PSTs with more mathematics education (38%). Furthermore, consistent with the EBS hypothesis, both groups of PSTs with greater mathematics education were in significantly greater agreement (average of 83% across the two groups) than BasicMath PSTs (66%) with the SPV that students’ symbolic reasoning precedes and serves as a necessary prerequisite to their verbal reasoning and story problem-solving abilities.

We believe that these empirical findings make a significant theoretical contribution to the study of pedagogical content knowledge and its relation to SMK and teachers’ decision-making practices.

**Tier 2: Teachers, Teaching, and the Classroom Context**

The primary work of Tier 2 researchers at UCB was to further understanding about the complexities of teaching algebra in the classroom context. In particular, Tier 2 researchers at UCB explored two primary themes. The first theme, teachers’ tolerance for discomfort, resulted in a paper that was published in January of 2004 in *The Journal for Curriculum and Supervision*. The theoretical lens that was developed in this paper is presently being used to frame ongoing studies of teacher uncertainty in professional development settings. A second paper on teachers’ discomfort is in progress and will be submitted for publication by the end of summer 2004. This paper builds on the former, elaborating in particular the ways in which professional development experiences for teachers should be cognizant of the various forms of pedagogical, cognitive, and belief-oriented discomfort teachers are likely to encounter as they adopt reform-based practices and curricular materials.
The second theme pursued by Tier 2 researchers at UCB involves the development of a framework for considering “turning points” in the work and thinking of middle school mathematics teachers. A paper elaborating a framework for understanding the catalysts that promote changes in middle school mathematics teachers’ beliefs and practices is under review by the *Journal for Research in Mathematics Education*. Follow-up data has been collected and will be analyzed during the summer of 2004. In particular, the framework established in this paper will be used to help define and elaborate ongoing professional development initiatives at UCB.

**Tier 2 / Tier 3: Summer 2003 Algebra Course and Fall 2003 Workshops**

Several articles describing and documenting the summer algebra course are currently under way. The main areas of focus for these articles include discourse, community, multiple representations of mathematical ideas and problem-solving strategies, teachers’ tolerance for discomfort, and a detailed description of the course itself. Several conference presentations based on data from the algebra course are to be delivered at the American Educational Research Association during the April 2004 annual meeting.

Analyses of the PD workshops and videotapes of lessons taught during the 2003-2004 academic year are also in progress. These analyses include constructing a detailed description of the PD sessions and patterns of teachers’ participation in these sessions, documenting the evolution of teachers’ pedagogical goals, and tracing pedagogical change in their videotaped lessons,

Initial findings, based on our ongoing analyses of data from the summer course, include the following:

1. The development of a sense of community and trust among teachers was fostered by activities such as solving problems collaboratively in small groups, building on one another’s ideas, and explaining and clarifying confusions as well as understandings of mathematical problems and solutions.
2. Modeling by the PD providers, and systematic placement of participants into small groups, also facilitated community development.
3. Teachers used a greater number of problem-solving strategies on the post-course assessment than on the pre-course assessment.

Initial analyses of lessons videotaped during the 2003-2004 academic year indicate that most teachers are attempting to incorporate into their teaching the ideas learned in the summer course and PD workshops. For example, several of the teachers have increased their use of problem-solving tasks and small group participation structures. Students are being asked to explain their thinking and problem-solving strategies to one another and the teacher, and the teachers are listening to students’ explanations in order to understand their reasoning processes.
OPPORTUNITIES FOR TRAINING AND DEVELOPMENT

Professional Development Opportunities for Middle School Teachers

The teacher professional development pilot study with five teachers from the Denver area provides strong encouragement for a model of belief confrontation and change on an important aspect of student thinking. We hope to extend this approach more broadly and test it with participants on the new Web page version.

Sixteen teachers in the summer 2003 algebra course examined, challenged, and improved their content knowledge through rich explorations of informal, pre-formal, and formal algebraic strategies. The course provided a solid foundation for their own knowledge development and provided skills for recognizing, appreciating, and building upon the kinds of knowledge and strategies their students bring to the algebra class.

The fall 2003 workshops have provided an opportunity for teachers to explore ways of incorporating into their teaching their newly acquired knowledge and perspectives on algebraic reasoning, in addition to developing skills and strategies for improving pedagogy.

Training Opportunities for Doctoral Students and Post-Doctoral Fellows

This project is providing opportunities for doctoral students and post-doctoral fellows to develop their expertise as professional development providers and researchers. The entire Tier 2 / Tier 3 team (consisting of 2 PIs, 4 doctoral students, and 2 post-doctoral fellows) participates in planning and debriefing meetings for each of the PD workshops. Three doctoral students have primary responsibilities for the conduct of the summer course and PD workshops. The other doctoral students and the post-doctoral fellows collect data during these workshops, and all team members are involved in videotaping and interviewing the teachers. The entire team also worked collaboratively to identify the several strands of data analysis that are currently under way, and each of the doctoral students and post-doctoral fellows is involved in conducting one or more of these analyses.

One doctoral student on the Tier 2 / Tier 3 team presented at the AERA 2003 annual meeting, and two are to present at the AERA 2004 annual meeting. One of the post-doctoral fellows has a presentation accepted at the annual meeting of the International Group for the Psychology of Mathematics Education in summer 2004.

OUTREACH ACTIVITIES

Outreach activities include exploring methods of expanding the workshops for the 2004-2005 school year in order to reach more teachers. We are particularly interested in using Internet and video-based PD strategies to make opportunities for improving algebra content knowledge and pedagogical skills widely available to middle school teachers, and we continue to probe for new possibilities through our ongoing communication with colleagues in this field.
CONTRIBUTIONS TO THE PHYSICAL DISCIPLINES OF THE PROJECT

The goals of developing detailed accounts of student learning and teacher knowledge and instructional practices in classroom contexts are central to a well-specified approach to teacher professional development and to newly emerging approaches of algebra instruction in the elementary and middle school grades.

CONTRIBUTIONS TO SCIENCE OR ENGINEERING

This research program on teacher knowledge and beliefs has implications that go beyond middle grade mathematics instruction. For example, in college-level physics education there is a recognized tension between those advocating instruction in a “top down” fashion, starting from scientific principles and moving to technological application, and those with a “bottom up” approach that uses technology as a basis to induce general scientific principles (Nathan, 2003a). This general bias for teaching from a formalisms-first perspective directly parallels the symbol precedence view exhibited by algebra teachers and suggests that this is a deep-seated and universal perspective on conceptual development held by educators, and perhaps by society as a whole. That this view has not been empirically established as the best method for teaching is important. That it challenges some available data on student math learning suggests that these views may actually hold some students back. The importance for SMET education is significant, and so we plan to explore these views more broadly among the science and mathematics community at the secondary and higher educational levels.

CONTRIBUTIONS TO DEVELOPMENT OF HUMAN RESOURCES

Middle school teachers participating in our summer 2003 algebra course and fall 2003 workshops represent a cross section of Colorado school districts. Future plans also include actively exploring methods of expanding the workshops for the 2004-2005 school year in order to reach more teachers. Several teachers have also expressed an interest in working with the Tier 2 / Tier 3 team to offer professional development to other teachers within their individual schools. We are exploring the feasibility of various options for addressing this request.

The experiences that we are providing to doctoral students and post-doctoral fellows will also help to develop the next generation of educational researchers. For example, the person who held a post-doctoral research position through summer 2003 is currently working on a project to develop multimedia cases of mathematics instruction. One of the persons currently holding a post-doctoral position has accepted a faculty position at a major university and plans to continue conducting research on teaching and teacher professional development, building on the work she has done on our project.

CONTRIBUTIONS TO INFRASTRUCTURE FOR RESEARCH AND EDUCATION

Teacher participants in the summer 2003 algebra course accessed the STELLAR Web site, which housed journal articles and other relevant documents along with a discussion board. This online component was intended to reinforce the community teachers created in the classroom and provide an opportunity to access that community outside the classroom.
Encouraged to take on the role of teacher-researchers, the teachers accept a shared responsibility for improving their classroom instruction and for putting into practice new ideas and skills they learn related to algebra and algebra pedagogy. Additional goals include creating and sustaining a teacher community or network, influencing teachers’ beliefs about algebra and pedagogy.

**CONTRIBUTIONS BEYOND SCIENCE AND ENGINEERING**

The No Child Left Behind legislation stipulates that high-quality professional development be made available to all K-12 public school teachers. Our efforts to develop an approach to PD that can be used with large numbers of teachers will contribute to this agenda.

Research on learning and teaching, as well as policies regarding teacher preparation, must include the drawbacks as well as the merits of teachers’ subject area knowledge. Recent reports have made much of the deficits in teacher subject-matter knowledge and their apparent impact on student learning and performance on high-stakes assessments (e.g., Educational Trust, 2002; Gonzales et al., 2000). Expert blind spot research suggests that teacher education and professional development programs must keep sight of the importance of pedagogical content knowledge in teaching. This emphasis must not be traded for attention to subject-matter preparation that can contribute to teachers’ holding inaccurate views of students’ intellectual development.

**PAPERS AND TALKS RESULTING FROM THIS PROJECT**

*Articles*


*Presentations and Talks*


Nathan, M. J. (2003, June). Technology’s other role in SMET education: How technology design and use can support science and mathematics education (and why we tend to dismiss such approaches). Northwestern University, School of Education and Social Policy.


REFERENCES


