Strategic interaction over mobile public bads

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Abstract
We consider analytically the non-cooperative behavior of $I$ private property owners who each controls the stock of a public bad such as invasive species, fire, or agricultural pests. The stock of the public bad can grow and disperse across the spatial domain. In this setting, we characterize the conditions under which private property owners will control or eradicate, and determine how this decision depends on property-specific features and on the behavior of other landowners. We show that high mobility or lower control by others result in lower private control. But when damage caused by the bad is sufficiently large, we find that complete eradication may be privately optimal (despite the lack of consideration of others’ welfare) – in these cases, eradication arises in the non-cooperative game and is also socially optimal. Finally, when property harboring the bad is not owned, or is owned in common, we derive the side payments required to efficiently control the mobile public bad.

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1 Introduction
We study the problem of controlling a public bad resource stock that is capable of growth and spread. Examples of such resources include fire, invasive species, insect pests, antibacterial resistance, and agricultural pests. The stock of the bad on property $i$ imposes damage to the owner of property $i$, which motivates the need for some level of control. But the problem is worsened by the mobility and renewability of the resource. If it is not eradicated, it will grow and disperse, thus imposing future damage across the spatial domain. Because it is a ubiquitous challenge with real-world policy implications, and is a tractable problem of the spatial allocation of scarce resources, this general problem has attracted the attention of economists. Early contributions focused on a single welfare-maximizing central planner who sought to optimally control a pest population (Lichtenberg and Zilberman 1986; Archer and Shogren 1996) or other biological invasions (Shogren 2000; Olson and Roy 2002).
While this early literature helped illuminate the efficient control by a social planner, it does not explicitly inform the positive or normative issues that arise when individual property owners (rather than a central planner) each make decentralized decisions about control on their own property. In that setting, damage caused by the public bad on property \( i \) motivates the owner of property \( i \) to undertake some level of private control of the bad. But any stock left uncontrolled will grow and disperse to other properties in subsequent periods. These dynamics introduce an externality and induce a spatial dynamic game between landowners under which owners engage in insufficiently low levels of control relative to what would be desired by the social planner. This paper thus concerns the private management of spatially-distributed, mobile public bads. We focus on deriving the biological and economic conditions under which private property owners will find it privately optimal to control or eradicate the public bad; and how those decentralized decisions depend on property-specific features and on the control decisions of others. A corollary is whether individual decisions ever lead to socially optimal management. The entire analysis is analytical, so we seek general insights that can inform both positive and normative aspects of this empirically-extensive class of challenges.

Two main strands of resource economics literature contribute to this question. First, it has become common recently to examine the optimal management of a species that is spatially distributed across the landscape. In that setting, a sole owner accounts for all spatial connections and optimizes her control efforts across space. The main purpose of this literature is to characterize the optimal design of policies to improve social welfare across the spatial domain. Because that problem grows in complexity with the spatial domain, most papers conduct numerical simulations either in stylized systems or in systems loosely parameterized by empirical observations. Leung et al. (2002) focus on the question of prevention vs. control for zebra mussels and find, for that species, that ex-ante prevention is more efficient than ex-post control. Burnett et al. (2007) apply a spatially-optimal control to compute the population levels of Miconia in Hawaii. Finnoff et al. (2010) use numerical simulations to contrast long-run solutions from an optimal control system and solutions from a static optimization problem. After having theoretically characterized the paths of expenditures and damages, Burnett et al. (2008) use the real-case of the Brown Tree snake in Hawai‘i to analyze the optimal integrated management of prevention and control. Kaiser and Burnett (2010) also use these data to apply their model for early detection and rapid response policies. Epanchin-Niell and Wilen (2012) develop a spatial numerical optimization model to derive the most efficient application of control effort across space. And Albers et al. (2010) develop a numerical model of trade to calculate the welfare loss associated with uniform, rather than spatially-optimized, policy. While some purely theoretical work examines a similar set of issues (e.g. Costello and Polasky (2008) and Blackwood et al. (2010)), those papers still focus on the optimal control by a central planner.

The second strand of this literature acknowledges the non-cooperative nature of private property owners, but conducts numerical analysis of particular systems. These studies account for the fact that control in one location may depend on others’ decisions and emphasize that decentralized responses to invasive spread is insufficient to manage it at the socially optimal level. Bhat and Huffaker (2007) consider a two patch control of beaver populations, and derive new mechanisms that can be used to induce cooperation in control efforts across the two patches. Grimsrud et al. (2008) analyze the impact of interaction
between two agents managing their own plots of land and apply their model to two different levels of infestation of Yellow Starthistle, an invasive plant in New Mexico. They show that coordination is required to minimize environmental damages, which is more likely with low levels of invasion. More recently, Fenichel et al. (2014) numerically analyze the influence of market-based instruments and that of the behavior of other citrus producers on private incentives for controlling invasive pests. They also discuss the role of spatial connectivity, which may erode property value and thus decrease private incentives for pest control.

While informative, and suggestive of mechanisms that may be applied more broadly, these papers do not provide theoretical findings so general lessons, and the conditions under which they arise, are difficult to extract. Moreover, the vast majority of this literature ignores heterogeneity in cost, damages, and dispersal rates, yet these sources of heterogeneity may significantly alter individual landowner incentives over control or cooperation. These incentives may also depend strategically on the decisions of adjacent owners, suggesting that homogeneity may be a restrictive assumption when characterizing the equilibrium of non-cooperative decisions by spatially-connected landowners. The aim of this paper is to analyze the private management of a spatially-distributed mobile public bad, and to examine the game theoretical interactions among non-cooperative land owners in order to derive general results such as the conditions under which control and eradication will emerge, those under which non-cooperation results in a socially optimal pattern of control, and the effects of system characteristics (e.g. the rate of spread) on non-cooperative outcomes.

More specifically, we develop an analytical model with an arbitrary number of spatially-distinct properties and discrete-time resource dynamics to analyze decentralized owners’ incentives and the equilibrium behavior across those owners. In our theoretical model we will also solve for the social planner’s optimal control pattern across space and time. While we think this as a contribution in its own right, we regard it primarily as a benchmark case against which to compare decentralized equilibria across non-coordinating property owners. We make three main analytical contributions. First, we show that the private trade-off between controlling the expansion of public bad on one’s own property and eradicating it depends on the magnitude of its spread. Furthermore, complete eradication is driven by the magnitude of patch connectivity. Second, in general we find the intuitive result that non-cooperative property owners will provide too little control of the public bad. This result is intuitive because private property owners will consider only their local costs and benefits of control, but will disregard the consequences of their actions on adjacent owners. A more nuanced result is to show analytically how the extent of this externality is driven by heterogeneity and other features of the problem. When damage inflicted by the stock is low, neither the social planner nor the non-cooperative private property owners will engage in much control, so little is to be gained from cooperation among private owners. In that case, private property delivers a near first-best outcome. But as the size of the damage increases, private property owners increase their control, but not as much as the social planner would have liked. Thus, as damage grows, so does society’s benefit from cooperation among property owners. This intuitive finding suggests that as the size of the externality grows, so does the importance of government intervention (or private ordering) to internalize the externality. But we find that this result only holds for moderate levels of damage. If damage grows enough, then private property owners will eradicate on their own property. We show that when this arises in the non-cooperative game, then it is also socially efficient.
Thus, if damage is sufficiently large, the cooperative and non-cooperative solutions converge, and there is no value to government intervention. This third contribution suggests that government intervention may be justified (to coordinate the actions of private land owners), but only in cases of intermediate damage. Our final contribution is to completely characterize the side payments that are necessary and sufficient to induce cooperative behavior among the non-cooperative property rights holders. Naturally, to the extent that properties are heterogeneous, these side payments will differ across space. We derive the magnitude of these side payments as a function of damage, cost, spread, and growth.

We organize the paper as follows: The analytical model is introduced in Section 2 and we derive the equilibrium strategies of non-cooperative property owners in Section 3. The social planner’s problem is introduced and solved in Section 4, which puts us in a position to compare the decentralized solution with the social planner’s. The possibility of eradication is analyzed in Section 5 and we calculate the cooperation-inducing side payments in Section 6. We conclude in Section 7. All proofs are found in the Appendix.

2 Model

The stock of a renewable public bad is spatially distributed. Space is divided into a set of $I$ mutually exclusive and exhaustive properties, each of which is assumed to be owned by a single profit maximizing owner. The stock residing in property $i$ at the beginning of time period $t$ is given by $x_{it}$ and control efforts undertaken in property $i$ will reduce the stock over the course of that time period. We denote the amount of stock removed in property $i$ by $h_{it}$, which leaves a “residual stock” at the end of the period of $e_{it} \equiv x_{it} - h_{it}$. The residual stock grows according to a growth function $g(e_{it})$, and the resource stock is distributed across the landscape. The fraction of the resource stock that moves from property $j$ to property $i$ is given by $D_{ji}$ (so $\sum_i D_{ji} \leq 1$), and the equation of motion of the resource stock is:

$$x_{it+1} = \sum_j D_{ji} g(e_{jt});$$

(1)

The resource stock in property $i$ imposes damage on owner $i$, and the damage function may be property-specific (for example, a weed may cause more damage in an agricultural area than in an industrial area). If the resource stock in $i$ is $x_i$, the marginal damage in $i$ is $k_i(x_i)$, where $k_i'(x_i) > 0$. The cost of control may also be property-specific (for example removing invasive mussels may be simpler in shallower water). The marginal cost of control in a property will also depend on the stock size in that property. This is the so-called stock effect for which the marginal extraction cost is a decreasing function of the stock. We model the marginal control cost as $c_i(x_i)$, where $c_i'(x_i) < 0$. Taking all relevant economic variables into account, the period-$t$ cost to owner $i$ of stock, $x_{it}$, and control, $h_{it}$ is:

$$\Phi_i(x_{it}, h_{it}) = \int_0^{x_{it} - h_{it}} k_i(s)ds + \int_{x_{it} - h_{it}}^{x_{it}} c_i(s)ds.$$

(2)

Invoking the identity $e_{it} \equiv x_{it} - h_{it}$, we can write the equation (3) as:

$$\Phi_i(x_{it}, e_{it}) = \int_0^{e_{it}} k_i(s)ds + \int_{e_{it}}^{x_{it}} c_i(s)ds.$$

(3)

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1We assume the usual conditions: $g(0) = 0$, $g'(e) > 0$, and $g''(e) < 0$. 4
The first term on the right hand side of Equation (3) is the total damage cost in property \( i \) during period \( t \) and the second term is the total cost of control. Equations (1) and (3) will be used extensively as they represent the payoffs and resource dynamics for this problem. While the payoff to owner \( i \) depends only on the stock and control in property \( i \), the stock itself will depend on past decisions in all other properties because the resource can move across space (see Equation 1). Thus, all properties are linked together and this is the sense in which we call this resource a \textit{public bad}.

3 Property owners’ control strategies

To begin, we assume that each of the \( I \) property owners makes a unilateral decision about how much control to engage in each period. This is a complicated decision for owner \( i \) for two reasons. First, owner \( i \)’s strategy about how much to control may depend on the strategies applied by all other agents. Second, because the resource grows and moves, owner \( i \)’s strategy must account for the fact that less control today implies higher growth and thus higher damage in future periods. In this section we derive the dynamic control strategies of each agent and use this to characterize the system-wide control, stock, and payoffs that are expected in this decentralized property right system.

3.1 Individual control strategies

In this setting all owners simultaneously choose the level of control, \( h_{it} \). Equivalently, they choose \( e_{it} \) (since \( e_{it} \equiv x_{it} - h_{it} \) and \( x_{it} \) is known at the time of the decision). Indeed, it turns out to be more mathematically convenient to keep track of the residual stock \( (e_{it}) \) rather than the explicit control \( (h_{it}) \), so we use \( e_{it} \) as the control variable for property owner \( i \). The explicit amount of control \( h_{it} \) can then simply be backed out. When owner \( i \) enters period \( t \), she observes \( x_{it} \) and chooses \( e_{it} \) to minimize her present value cost taking all other variables as given, as follows:

\[
\min_{e_{it} \geq 0} (\Phi_i(x_{it}, e_{it}) + \delta J_{t+1}(x_{it+1}))
\]

where the function \( J_{t+1}(x_{it+1}) \) captures the continuation payoff as a function of the future resource stock (which will, itself, be a function of others’ decisions). This is subject to Equation 1 which defines the state transitions as a function of all owners’ controls. Focusing on an interior equilibrium (for which \( e_{jt} > 0 \ \forall j, t \), we can immediately write down the solution to this \( I \)-dimensional dynamic game, which we summarize as follows:

**Proposition 1.** The equilibrium of the \( I \)-property public bad dynamic game is characterized by residual stock in property \( i \) \( (\hat{e}_{it}) \) given as follows:

\[
k_i(\hat{e}_{it}) = c_i(\hat{e}_{it}) - \delta c_i(\hat{x}_{it+1})D_{it}g'(\hat{e}_{it})
\]

Here, \( \hat{e}_{it} \) and \( \hat{x}_{it} \) denote the residual stock and resource stock in property \( i \), respectively. The level of control is simply \( \hat{h}_{it} = \hat{x}_{it} - \hat{e}_{it} \). Proposition 1 shows that residual stock results from a trade-off between the current marginal damage (on the LHS) and the long-run marginal control cost (on the RHS). Owner \( i \) will control the bad until the current marginal
damage is equal to the current marginal cost of removing one additional unit of the stock, mitigated by the discounted future cost implied by an increased stock.

We note also that the strategy of owner $i$ depends on $\hat{x}_{it+1}$, which suggests that owner $i$’s decision will depend on the decisions of other owners $j$ for whom $D_{ji} \neq 0$ (see Equation 1). If an adjacent owner engages in less control (and so leaves a larger $\hat{e}_{jt}$), how will owner $i$ respond? We find a kind of “race to the bottom” emerges, in which less control by owner $j$ implies less control for all connected owners. This consequence of strategic dynamic interactions among property owners is formalized as follows:

**Proposition 2.** There is a “race to the bottom” regarding decentralized control of the public bad by private owners: A larger residual stock in one property causes an increase in the optimal residual stock in all connected properties: $\frac{\partial \hat{e}_{it}}{\partial e_{it}} > 0$, where $D_{ij} \neq 0$.

Proposition 2 implies that owners’ choices are strategic complements; changes in public bad control on property $j$ gives the owner $i$ an incentive to act in the same manner. The strategic nature of interactions among property owners implies a tragedy of the commons: the strategic reaction to each others’ decisions may induce a kind of domino effect which facilitates the spread of a spatially mobile public bad. This is a spatial analog of the weakest link problem because lower control by a single owner eventually leads to a loss in welfare across the entire spatial domain. While Proposition 1 characterizes the strategies of each owner, we next focus on the role of spatial spread on individual strategies.

### 3.2 How does “spread” affect individual strategies?

One of the distinguishing features of this analysis is that the public bad can both grow and spread. Without spread, this reduces to a relatively straightforward optimal control problem for a single planner. But in the presence of spread, the degree to which owner $i$’s strategy depends on decisions by owner $j$ will depend on the magnitude of spatial connectivity between cells in addition to strategic interactions. Here we examine the impact of the spread parameters in order to identify how changes in control on property $i$ are influenced by changes dispersal patterns. To sharpen the analysis we focus on the two-property ($I = 2$) case. In that case, there are two self-retention parameters: $D_{ii}$, and $D_{jj}$, and two dispersal parameters: $D_{ij}$, and $D_{ji}$.

The impact of self-retention of property $i$, $D_{ii}$, and the spread from patch $j$ to $i$, $D_{ji}$ imply a higher quantity of the public bad. A more nuanced question is whether owner $i$’s residual stock will depend on what happens in patch $j$, that is the in-property rate, $D_{jj}$, and the spread from $i$ to $j$. This will affect residual stock in patch $j$, and thus due to strategic interactions, will indirectly affects residual stock in patch $i$. More specifically, if $j$ responds by engaging in less control, then by Proposition 2, owner $i$ may also respond by engaging in less control. All results on the dependence of owner $i$’s residual stock on the spread of the public bad are summarized as follows:

**Proposition 3.** Assume $D_{ii}$ and $D_{jj}$ are sufficiently high. Then an increase in off-property spread (either $D_{ij}$ or $D_{ij}$), results in a larger residual stock level in property $i$:

$$\frac{\partial \hat{e}_{i}}{\partial D_{ji}} > 0; \quad \frac{\partial \hat{e}_{i}}{\partial D_{ij}} > 0.$$
A higher value of self-retention rate, respectively $D_{ii}$ and $D_{jj}$, results in a lower residual stock level in property $i$ if and only if the respective marginal cost is inelastic:

$$\frac{\partial \hat{e}_i}{\partial D_{ii}} < 0 \iff 1 > \varepsilon_1 \quad \text{with} \quad \varepsilon_1 = -D_{ii}g(\hat{e}_i)\frac{c'_i(\hat{x}_{it+1})}{c_i(\hat{x}_{it+1})} > 0$$

$$\frac{\partial \hat{e}_i}{\partial D_{jj}} < 0 \iff 1 > \varepsilon_2 \quad \text{with} \quad \varepsilon_2 = -D_{jj}g(\hat{e}_j)\frac{c'_j(\hat{x}_{jt+1})}{c_j(\hat{x}_{jt+1})} > 0$$

Proposition 3 shows that both spread parameters increase the residual stock in patch $i$. An increase in $D_{ji}$ seem to be as if owner $j$ now engages in less control, since more resource moves toward patch $i$. Consequently, this effect results directly from the spatial connectivity between patches, and pushes owner $i$ to decrease her control in her own cell. The effect of $D_{ij}$ is more surprising since it describes a higher movement from $i$ to $j$ which may suggest a decrease in the residual stock of patch $i$. But, because of strategic interactions between agents, following Proposition 2, this will incentivize agent $i$ to decrease her control in her own patch.

Analyzing the effects of self-retention ($D_{ii}$ and $D_{jj}$) also yields insights, though this becomes more complicated. Here, whether $\hat{e}_{it}$ will increase or decrease in response to a rise in $D_{ii}$ (or $D_{jj}$) will depend on the nature of the costs of control. If the marginal cost of control is relatively flat (so $c'(.) \approx 0$), then owner $i$ will engage in more control if $D_{ii}$ is larger. This makes intuitive sense: If a pest population is more likely to persist on one’s property, then it seems intuitive that the owner would engage in more control compared to a case in which it is likely to quickly move off of one’s property. As a consequence of strategic dynamic interactions, a similar result emerges regarding owner $i$’s response to an increase in $D_{jj}$. But these results can be flipped if marginal cost is sufficiently steep. We next contrast the behavior of individual property owners with that of the social planner.

4 The social planner

The spatial dynamic game reveals that private agents ignore the spatial connectivity between properties. The tragedy of the commons should emerge from this situation, that is each owner seems to engage in insufficiently low control of the public bad. To verify this intuition, and to explore the conditions under which this is indeed the case (or whether the result externality is large or small), requires solving the benchmark case in which the entire spatial domain is managed by a sole owner. The sole owner must optimize the spatial and temporal control to minimize the present value of the sum of costs to all properties, subject to the resource dynamics. Written as a dynamic programming equation, the sole owner’s problem is:

$$V_t(x_t) = \min_{\{e_{1t}, e_{2t}, \ldots, e_{Nt}\} \geq 0} \sum_i \Phi(x_{it}, e_{it}) + \delta V_{t+1}(x_{t+1})$$

subject to the Equation (1), and where the bold notation $x_t$ indicates the vector $x_t \equiv [x_{1t}, x_{2t}, \ldots, x_{Nt}]$. This appears to be an incredibly complicated problem to solve, particularly as $I$ gets large, because it involves an $I$ dimensional decision where each decision is connected over time via the spread and growth dynamics. But it turns out that this problem can be solved analytically, and that the optimal spatial-temporal control policy can be completely characterized. For an interior solution, the optimal policy is given as follows:
Proposition 4. The sole owner’s optimal control strategy has residual stocks, \( \bar{e}_t > 0 \), characterized as follows

\[
k_i(\bar{e}_{it}) = c_i(\bar{e}_{it}) - \delta \sum_j c_j(\bar{x}_{jt+1}) D_{ij} g'(\bar{e}_{it}).
\] (7)

In a manner similar to the decentralized result (Proposition 1), the sole owner’s optimal residual stock results from a trade-off between marginal damage (on the LHS) and the marginal control cost (on the RHS). Again, the marginal cost of control is composed by the current marginal control cost and the sum of the discounted marginal control cost in the future.

A key focus of this section is to use this result to compare the equilibrium of the decentralized game with the socially optimal (i.e. sole owner’s) level of control. Several intuitive implications emerge: First, consider the case in which \( D_{ii} = 1 \) (so \( D_{ji} = 0 \) for all \( j \neq i \)). Then no spatial externality exists because owner \( i \) is a sole owner who is completely unconnected to the outside world. In this case, we would expect the decentralized solution to equal the socially optimal solution, so \( \hat{e}_i = \bar{e}_i \). More generally though, if \( D_{ji} \neq 0 \), then an externality exists and we would expect the decentralized property owners to engage in insufficiently low control. Indeed, this is what we find and our results are summarized as follows:

Proposition 5. In any period \( t \), the property-\( i \) residual stock that emerges from non-cooperation (\( \hat{e}_{it} \)) and the property-\( i \) residual stock that emerges from a sole owner’s decision (\( \bar{e}_{it} \)) are compared as follows:

(a) In the absence of spread (so \( D_{ij} = 0 \) \( \forall i \neq j \)), the Nash equilibrium is equivalent to the socially optimal policy for all properties, \( \hat{e}_{it} = \bar{e}_{it} \).

(b) When property \( i \) is a pure source, i.e. \( D_{ii} = 0 \), the Nash equilibrium is strictly higher than the optimal policy for each property, \( \hat{e}_{it} > \bar{e}_{it} \).

(c) When \( D_{ii} \in (0, 1) \), we have, for any property \( i \), \( \hat{e}_{it} \geq \bar{e}_{it} \).

Proposition 5 confirms the tragedy of commons emerges with private management, except obviously when there is no spatial connectivity. Otherwise, as soon as an area behaves as a source, i.e. augments the population in adjacent area, controlling the public bad under laissez-faire is insufficient by comparison to the social outcome.

It is now interesting to tackle this question when eradication makes sense. Indeed, this strategy suggests a higher degree of intervention. It is consequently relevant to wonder if the situations for which such an intervention under private is feasible is close to conditions under social management.

5 The emergence of eradication

While it is not common to analyze eradication in dynamic resource models, it may be appropriate when dealing with a public bad. Naturally, though, the decision of whether to completely eradicate the a public bad such as pests, invasive species or epidemics will depend on adjacent owners’ actions because if they lack control on their property, the likelihood
of future infestation may be very high. Strategic interactions should be once again an important driver of individual decision. Thus, the purpose of this section is to analyze the circumstances under which eradication is either a socially efficient outcome and/or an outcome of decentralized decisions by property owners. This will illuminate circumstances under which there is consistency between cooperative and non-cooperative behaviors in which case no market manipulation will be required to achieve efficiency.

We begin by analyzing the case in which full eradication is a socially-optimal policy for the spatially-connected renewable public bad. At first glance this seems to be a daunting task for a social planner - she must account for all spatial connections, costs of control, and heterogeneous damage functions to ultimately determine whether it is optimal to eradicate the resource from all spatial properties. If she eradicates in only a subset of properties, the resource will grow and redistribute itself in the future. On the other hand, complete eradication is forever - complete eradication reduces all future damage and control costs to zero.

Recall that Proposition 4 spells out the optimal residual stock level to which the social planner would like to control in each period. If this condition can be met in all properties, then eradication will not be an optimal strategy. But, for example, suppose the marginal damage is very high (even for a small stock). In that case, even if residual stock is reduced to zero, the condition cannot be met. We will show in Proposition 6 part (a), that if this is the case, it is optimal to eradicate the entire resource stock in all properties.

A similar analysis can be used to determine whether complete eradication will arise out of non-cooperative behavior across property owners. Any given property owner \( i \) would like to solve Equation 5 in Proposition 1. But if this is not possible (again, consider the case in which even reducing residual stock to near zero still entails very high marginal damage), it turns out that the optimal decision for agent \( i \) is to completely eradicate the stock on her property. If this happens to be the case for all property owners, then complete eradication across the entire spatial domain will arise out of non-cooperative behavior. These results are summarized as follows:

**Proposition 6.** Complete eradication across the entire spatial domain:

(a) Is socially optimal if and only if:

\[
\min_{i \in I} \left[ k_i(0) - (1 - \delta D_{ii} g'(0)) c_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij} c_j(0) \right] > 0. \tag{8}
\]

(b) Arises from non-cooperative behavior of property owners if and only if:

\[
\min_{i \in I} \left[ k_i(0) - (1 - \delta D_{ii} g'(0)) c_i(0) \right] > 0. \tag{9}
\]

It turns out that realistic cases exist in which full eradication in fact does arise, as is suggested by Proposition 6.

Contrary to control policy, in case of complete eradication, it is irrelevant to compare the equilibrium of the decentralized game with the socially optimal level of control since, \( \bar{e} = \hat{e} = 0 \). However, this does not imply that there is consistency between the decentralized and socially optimal solutions since conditions for which eradication may occur differ. We can nevertheless confirm the common intuition:
Proposition 7. If complete eradication emerges as a decentralized solution, then complete eradication is a socially optimal outcome.

Proposition 7 shows that it is possible that there is consistency between the optimal control by decentralized private owners and the optimal control by a social planner. If decentralized property owners all find it privately optimal to eradicate the public bad (presumably because the damages they faced were sufficiently large to justify the cost of eradication), then complete spatial eradication is also socially optimal. The consistency is straightforward if \( 1 < \delta D_{ii} g'(0) \). In other words, if the stock increase in any cell \( i \) generating by the reproduction of the first individual of the population in that area, \( D_{ii} g'(0) \) is high enough, eradication may arise under centralized or decentralized management. This thus implies the following corollary:

Corollary 1. If self retention is sufficiently high on all properties, that is:

\[
\min_{i \in I} D_{ii} \geq \frac{1}{\delta g'(0)},
\]

then complete eradication is socially optimal and will emerge from non-cooperative behavior.

Corollary 1 outlines that there exists a threshold value of self-retention such that there will be no tension between socially optimal and private incentives if the lower self-retention rate among all patches is higher than this threshold value.

Conversely, it may often be the case that complete eradication is socially optimal, but will not arise from decentralized owners’ decisions. More specifically, it could be the case either if all agents privately opt for control policy, or if eradication arise on some properties, but not on others. For example, this latter situation may arise if property \( i \) has high marginal costs of fire (perhaps it has an expensive home on it) and property \( j \) has low marginal costs of fire (perhaps it is rangeland), then this analysis suggests that owner \( i \) may find it privately optimal to extinguish the fire on her property while owner \( j \) does not. But even in that case (when one decentralized owner eradicates and another does not), it may be socially optimal to fully eradicate on all properties (for example if \( i \) is downwind of \( j \)). This result could arise because failing to eradicate on parcel \( j \) eventually causes damage on parcel \( i \). These situations are summarized in the following corollary:

Proposition 8. Complete eradication can be the socially optimal policy if, \( \forall i \) satisfying inequality (8), while a private management can lead to:

(i) Partial eradication if the following conditions holds:

\[
\min_{i \in I} [k_i(0) - c_i(0) (1 - \delta D_{ii} g'(0))] \leq 0
\]

(ii) No eradication if the following conditions holds:

\[
\max_{i \in I} [k_i(0) - c_i(0) (1 - \delta D_{ii} g'(0))] \leq 0
\]
is socially optimal to eradicate in all cells. This will enable us to have an in-depth discussion on conditions for which eradication is biologically and/or economically feasible. Indeed, necessary conditions for this to occur are first that condition \((10)\) is not met. It is sufficient that there exists a property \(i\) such that we observe the following:

\[
D_{ii} < \frac{1}{\delta g'(0)}.
\]  

(13)

Second, the marginal abatement cost (at small resource stock) is neither too low nor too high:

\[
\frac{k_i(0)}{1 - \delta D_{ii}g'(0)} \leq c_i(0) < \frac{k_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij}c_j(0)}{1 - \delta D_{ii}g'(0)}.
\]  

(14)

Conditions \((13)\) and \((14)\) yield interesting implications regarding the impact of spatial characteristics on the existing tensions between socially optimal and strategic behaviors. Indeed, we obtain that the self-retention rate in at least one patch is upper-bounded, and the range in which the current marginal cost for controlling the first individual must fall depends on in- and out-property dispersal rates of that patch. Assuming that these characteristics may change, conditions for which there may exist such a tension should be modified as follows:

**Proposition 9.** Suppose complete eradication is socially optimal, and denote by \(i\) a property with sufficiently low self-retention \((D_{ii} < \frac{1}{\delta g'(0)})\). Then the effect of spatial parameters on the emergence of eradication is summarized as follows:

1. An increase in dispersal \(D_{ij}\) (where \(i \neq j\)) makes the emergence of complete eradication less likely under decentralized management;

2. Provided that self retention remains lower than \(\frac{1}{\delta g'(0)}\), an increase in self retention makes the emergence of complete eradication less likely under decentralized management.

The effect of spread is straightforward. An increase in out-dispersal increases the incentive for the social planner to eradicate, but does not alter the incentives of decentralized property owners. This thus increase the opportunity to observe a tension between the control scenario and the *laissez-faire* situation. But the effect of self retention is harder to intuit. Corollary 1 and Proposition 9 imply that when all properties except one (say \(i\)) are characterized by sufficiently high values of self retention, then an increase in the value of self retention in property \(i\) may have two opposing effects. First, if the increase is such that self retention is now higher than the threshold value, then Corollary 1 implies that it has a positive effect as it removes the potential tension between socially optimal and private incentives. However, if the initial value is so low that the increase is not sufficient to move it over the threshold, then the effect is negative as it enlarges the set of cases where complete eradication does not emerge under decentralized management. These results can be used to assess whether one might expect strong consistency between socially and decentralized management or tensions arising due to strategic behavior among property owners.
6 Cooperation via side-payments

So far, we have contrasted the decentralized decisions of property owners who are harmed by a public bad that moves across space with the optimal solution of a social planner who can perfectly anticipate the spatial migration of the public bad and who can perfectly target control efforts across space. We have more particularly spelled out that the optimal policy of eradication may emerged spontaneously from, at least part of, decentralized property owners depending on spatial characteristics. This naturally begs the question of what institutions can move us from the decentralized solution to the socially optimal solution, and to which extent the mechanism is robust to spatial externalities. Particularly, the situation in which some property owners already eradicate in their own area provides an interesting room to examine the potential for voluntary mechanisms to achieve the socially-optimal level of control, that is complete eradication. Indeed, there is only few agents to incentivize to achieve cooperation. The focus of this section is to assess the impact of spatial characteristics on the size of the benefit from cooperation.

To sharpen the analysis, we focus on the simplest case of two properties (i and j). This purpose is to investigate if they’re may be better off when there is a monetary transfer from owner i to owner j compared to the (no-transfer) decentralized management outcome. While we have not explicitly modeled transaction costs, the larger is the potential benefit from cooperation, the more likely it is that transaction costs can be overcome. Thus, we would like to explore the conditions under which we might expect a large, or small, benefit from complete cooperation over the control of this spatially-mobile public bad. We first characterize the necessary and sufficient conditions for this partial eradication to occur in the following lemma:

Lemma 1. Assume that complete eradication is socially optimal. Now, under decentralized management the stock will be partially eradicated (that is, in the first property only) if and only if:

\[ k_j(0) - [1 - \delta D_{jj}g'(0)]c_j(0) < 0, \]
\[ k_i(0) - c_i(0) + \delta g'(0)D_{ij}c_i(D_{jj}g(\hat{e}_j)) > 0 \]

where \( \hat{e}_j > 0 \) is characterized implicitly by the following equality:

\[ k_j(\hat{e}_j) - c_j(\hat{e}_j) + \delta D_{jj}g'(\hat{e}_j)c_j(D_{jj}g(\hat{e}_j)) = 0. \]

Lemma 1 characterize the decisions of the two property owners. Conditions (15) and (17) highlight that agent j opt for a control policy leaving a residual stock \( \hat{e}_j > 0 \). Condition (16) emphasize that owner i is incited to eradicate stock over her space each time period. Furthermore, since \( c'(\cdot) < 0 \) and \( D_{ij} > 0 \), we observe that the incentives of owner i are weaker than under complete eradication defined by condition (9). Such a decentralized situation will generate a stream of minimized economic costs and damages \( \hat{\Pi}_i \) for agent i (who eradicate each time period) and respectively \( \bar{\Pi}_j \) for agent j (who controls the size of public bad), which are defined as follows:
\[ \hat{\Pi}_i = - \left[ \int_0^{x_0} k_i(s) ds + \int_0^{x_0} c_i(s) ds + \frac{\delta}{1-\delta} \left( \int_0^{x_0} k_i(s) ds + \int_0^{D_{ij}g(\hat{e}_j)} c_i(s) ds \right) \right] \] (18)

\[ \hat{\Pi}_j = - \left[ \int_0^{\hat{e}_j} k_j(s) ds + \int_{\hat{e}_j}^{x_0} c_j(s) ds + \frac{\delta}{1-\delta} \left( \int_0^{\hat{e}_j} k_j(s) ds + \int_{\hat{e}_j}^{D_{ij}g(\hat{e}_j)} c_j(s) ds \right) \right] \] (19)

Conversely, as the socially outcome is defined by complete eradication, the gains of each owner will be defined as:

\[ \bar{\Pi}_i = - \int_0^{x_0} c_i(s) ds \] (20)

\[ \bar{\Pi}_j = - \int_0^{x_0} c_j(s) ds \] (21)

Now, consider that owner \( i \) want to make a payment to owner \( j \) to leave no residual stock in his property. The payment will be feasible if there exists positive gains to cooperate, that is if the difference between the sum of cooperative gain (18)-(19) is higher than the sum of decentralized gains (20)-(21) as highlighted by the following proposition:

**Proposition 10.** Assume that there are two properties (\( i \) and \( j \)), that complete eradication is socially optimal (that is, the cooperative outcome), and that conditions (15), (16) and (17) are satisfied. Then, there are positive gains from cooperation.

Proposition 10 shows that, under decentralized (non cooperative) management, the manager of property \( i \) has incentives to compensate the other manager so that he would lower the residual stock level of the stock in his property. The value of this monetary transfer is however specific to the spatial characteristics of property in which there is no eradication. As such, higher immigration rate in the property under eradication should imply welfare losses (compared to situation with a small rate), so owner who eradicates should be more prone to pay a transfer. We summarize our findings as follows:

**Proposition 11.** Assume that there are two properties (\( i \) and \( j \)), that complete eradication is socially optimal (that is, the cooperative outcome), and that conditions (15), (16) and (17) are satisfied. Then, the following conclusions hold:

1. The gains from cooperation increase with an increase in the value of dispersal from property \( j \) to property \( i \) (parameter \( D_{ji} \)).

2. If the marginal cost is elastic, we observe that the higher the self-retention \( D_{jj} \), the higher the gains from cooperation.

Proposition 11 seems to suggest that compensation should be oriented toward the weakest link, that is, the property \( j \) for which:

\[ k_j(0) - (1 - \delta D_{jj}g'(0)) c_j(0) = \min_{i \in I} k_i(0) - (1 - \delta D_{ii}g'(0)) c_i(0). \]

An appropriate transfer payment might be used to induce owner \( j \) to engage in additional control, thus lowering his residual stock level, because it would benefit the adjacent owners. Moreover, the larger is dispersal from property \( j \) to others, the larger is the surplus resulting from potential cooperation, thus potentially increasing the willingness of others to compensate the weakest link.
7 Conclusion

We have developed and analyzed a model of a renewable resource public bad, such as an invasive species, fire, antibacterial resistance, or agricultural pest, that can move across space. Decentralized property owners undertake costly control to reduce damage on their own properties, and because the resource is mobile, this control has consequences for all other property owners. The resulting externality induces a spatial-temporal game between the property owners who will each act strategically given the behavior of other owners. Our first contribution is to completely characterize the equilibrium strategy of each owner and the resulting effects on stock and control of the public bad across space. We also solve for the socially optimal level of control across space and show that it always (weakly) exceeds the level of control undertaken by decentralized owners.

A key focus of our analysis is on the conditions under which eradication is undertaken by decentralized owners or is desirable by the social planner. We find that there is often consistency between these - realistic cases exist in which all decentralized owners will eradicate the stock on their property; in these cases the social planner would choose the same level of control, so no policy intervention is warranted. But cases also exist in which one or more decentralized owner fails to completely eradicate (even though it is socially desirable). In such cases, side-payments can induce appropriate control, and we have characterized the features of the problem that lead to large or small potential gains from this kind of side payment.

To obtain sharp analytical results of this spatial-temporal game has required making some simplifying assumptions. We modeled marginal damage in property \(i\) as a function of resource stock in property \(i\), which depends on the previous period’s stock in all properties and the spread from those properties to property \(i\). A more complicated version of damage would allow for damage in period \(t\) to depend on how much damage had been caused in previous periods. We modeled the marginal control cost as a decreasing function of the stock of the bad; the higher is the local stock of the bad, the smaller is the marginal cost of abatement. While this follows the literature and seems to fit most applications, an extension could allow for marginal cost to also depend explicitly on the quantity removed. Regarding the spread of the stock, we have assumed that the fraction of the stock that spreads from property \(i\) to property \(j\) is constant. An extension could allow for the spread to depend on the density of the stock in both areas. While these changes would complicate the solution to our model, we think they are unlikely to overturn the main findings of this paper. But these are fertile opportunities for empirical applications of this work.

Overall, our results suggest a peculiar result about the gain from coordination among decentralized property owners. If the damages or the magnitude of the externality are small, then decentralized owners choose a level of control that is lower than, but approximately equal to, the control that would be chosen by a social planner. In those cases, the gain from coordinated of decentralized owners is likely to be small. If damages and spread are moderate, an interior solution is likely to obtain under which some control will be undertaken by the decentralized property owners, but that this control will fall far short of what would be chosen by the social planner. In these cases, the gains from coordination are large. But when private damages are sufficiently large, decentralized owners will choose to eradicate on their own property. We proved that in those cases, complete eradication is also socially optimal.
In such cases, there is no gain from coordination. Taken together, these results suggest that the gain from coordination among decentralized owners is largest for an intermediate level of damage, which may run counter to intuition and may be suggestive of cases when government intervention or coordination schemes are most economically relevant.

References


8 Appendix

Proof of Proposition 1

The result follows immediately from the first order conditions.

Proof of Proposition 2

Since we consider a full game setting, we denote \( \Psi_i = k_i(e_i) - c_i(e_i) + \delta c_i(x_i)D_{ii}g'(e_i) = 0 \) the first order condition (FOC) which defines the best response of the manager in property \( i \), i.e. the set of best responses for different residual stock levels of the other agents in the other properties. Let \( e_i(e_j, ..., e_l) \) the reaction function of agent \( i \). According to the FOC, we know that \( \Psi_i[e_i(e_j, ..., e_l), e_j, ..., e_l] = 0 \).

Total differentiating this expression, we get:

\[
\frac{\partial \Psi_i}{\partial e_i} \frac{\partial e_i}{\partial e_j} + \frac{\partial \Psi_i}{\partial e_j} = 0 \quad \Leftrightarrow \quad \frac{\partial e_i}{\partial e_j} = -\frac{\partial \Psi_i}{\partial e_i} (22)
\]

with

\[
\frac{\partial \Psi_i}{\partial e_i} = k'_i(\hat{e}_i) - c'_i(\hat{e}_i) + \delta D_{ii} \left[ c_i(x_{it+1})g''(\hat{e}_i) + c'_i(x_{it+1})D_{ii}g'(\hat{e}_i) \right] (23)
\]

\[
\frac{\partial \Psi_i}{\partial e_j} = \delta D_{ij}g'(e_i)c'(x_i)D_{jj}g'(e_j) (24)
\]

Equation (23) is positive since it is the second order condition and equation (24) is negative since \( c'(.) < 0 \) and \( g'(.) > 0 \). So \( \frac{\partial e_i}{\partial e_j} > 0 \). This concludes the proof.

Proof of Proposition 3

In a case with two properties \( i \) and \( j \), assuming interior equilibria, we have:

\[
k_i(e_i) = c_i(e_i) - \delta c_i(x_i)D_{ii}g'(e_i) \quad (25)
\]

\[
k_j(e_j) = c_j(e_j) - \delta c_j(x_j)D_{jj}g'(e_j) \quad (26)
\]

By time and state independence, we can remove subscript \( t \) in expressions (25) and (26). They imply that \( e_j \) and \( e_l \) are the solution to the above system, which in turn implies that \( e_i \) and \( e_j \) are both functions of \( \theta = \{D_{ii}, D_{jj}, D_{ij}, D_{ji}\} \). These two first order conditions can thus be written as a function of the parameter, \( \theta \), as \( \Psi_i(e_i(\theta), e_j, \theta) = 0 \) and \( \Psi_j(e_i(\theta), e_j, \theta) = 0 \). We can thus totally differentiate both conditions:

\[
\begin{cases}
\frac{\partial \Psi_i}{\partial e_i} \frac{\partial e_i}{\partial \theta} + \frac{\partial \Psi_i}{\partial e_j} \frac{\partial e_j}{\partial \theta} + \frac{\partial \Psi_j}{\partial \theta} = 0 \\
\frac{\partial \Psi_j}{\partial e_i} \frac{\partial e_i}{\partial \theta} + \frac{\partial \Psi_j}{\partial e_j} \frac{\partial e_j}{\partial \theta} + \frac{\partial \Psi_i}{\partial \theta} = 0
\end{cases} \quad (27)
\]

Solving this system, we get that \( \frac{\partial e_i}{\partial \theta} \) (and symmetrically \( \frac{\partial e_j}{\partial \theta} \)):

\[
\frac{\partial e_i}{\partial \theta} = -\frac{\partial \Psi_i}{\partial \theta} \frac{\partial e_i}{\partial \theta} + \frac{\partial \Psi_i}{\partial \theta} \frac{\partial e_i}{\partial \theta} - \frac{\partial \Psi_j}{\partial \theta} \frac{\partial e_j}{\partial \theta}
\]

with

\[
\frac{\partial \Psi_i}{\partial e_i} = SOC_i > 0 \quad \frac{\partial \Psi_i}{\partial e_j} = \delta D_{ii}D_{ij}g'(e_i)g'(e_j)c'_i(x_i) < 0
\]

\[
\frac{\partial \Psi_j}{\partial e_i} = SOC_j > 0 \quad \frac{\partial \Psi_j}{\partial e_j} = \delta D_{jj}D_{ij}g'(e_i)g'(e_j)c'_j(x_j) < 0
\]
\( \theta \)

| \( D_{ii} \) | \( \frac{\partial \psi_i}{\partial D_{ii}} = \delta g'(e_i) \left[ c_i(x_i) + D_{ii} c_i'(x_i) g(e_i) \right] \) | \( \frac{\partial \psi_i}{\partial D_{ii}} = 0 \) |
| \( D_{ji} \) | \( \frac{\partial \psi_i}{\partial D_{ji}} = \delta D_{ij} g'(e_i) c_i'(x_i) g(e_j) \) | \( \frac{\partial \psi_j}{\partial D_{ji}} = 0 \) |
| \( D_{jj} \) | \( \frac{\partial \psi_k}{\partial D_{jj}} = 0 \) | \( \frac{\partial \psi_j}{\partial D_{jj}} = \delta g'(e_j) \left[ c_j(x_j) + D_{jj} c_j'(x_j) g(e_j) \right] \) |
| \( D_{ij} \) | \( \frac{\partial \psi_i}{\partial D_{ij}} = 0 \) | \( \frac{\partial \psi_j}{\partial D_{ij}} = \delta D_{jj} g'(e_j) c_j'(x_j) g(e_i) \) |

And the following derivatives:

Observe that \( \forall \theta \) the denominator of equation (28) is: \( SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} g'(e_i) g'(e_j) c_i'(x_i) c_j'(x_j) \).

If either \( D_{ii} \), or \( D_{jj} \) is sufficiently high, the denominator is positive.

Using derivatives in Table 2, we deduce the following derivatives:

\[
\frac{\partial e_i}{\partial D_{ii}} = \frac{\delta g'(e_i) \left[ c_i(x_i) + D_{ii} c_i'(x_i) g(e_i) \right] SOC_j}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} g'(e_i) g'(e_j) c_i'(x_i) c_j'(x_j)} \quad (29)
\]

\[
\frac{\partial e_i}{\partial D_{ji}} = -\frac{\delta D_{ij} g'(e_i) c_i'(x_i) SOC_j}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} g'(e_i) g'(e_j) c_i'(x_i) c_j'(x_j)} \quad (30)
\]

\[
\frac{\partial e_i}{\partial D_{jj}} = \frac{\delta^2 D_{ii} D_{jj} g'(e_i) c_i'(x_i)^2 c_j'(x_j) \left[ c_j(x_j) + D_{jj} c_j'(x_j) g(e_j) \right]}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} g'(e_i) g'(e_j) c_i'(x_i) c_j'(x_j)} \quad (31)
\]

\[
\frac{\partial e_i}{\partial D_{ij}} = \frac{\delta^2 D_{ii} D_{jj} D_{ij} g'(e_i) c_i'(x_i)^2 c_j'(x_j) g(e_j)}{SOC_i SOC_j - \delta^2 D_{ii} D_{jj} D_{ij} g'(e_i) g'(e_j) c_i'(x_i) c_j'(x_j)} \quad (32)
\]

Since the denominator is always the same, assuming it is positive, we easily observe that \( \frac{\partial e_i}{\partial D_{ii}} > 0 \) and \( \frac{\partial e_i}{\partial D_{jj}} > 0 \). We observe that is the marginal cost is quite inelastic, i.e. \( 1 > -D_{ii} g(e_i) c_i'(x_i) > 0 \), then \( \frac{\partial e_i}{\partial D_{ii}} < 0 \). The sign of \( \frac{\partial e_i}{\partial D_{ij}} \) depends also on the marginal cost elasticity. If the marginal cost is quite inelastic, i.e. \( 1 > -D_{jj} g(e_j) c_j'(x_j) > 0 \), then \( \frac{\partial e_i}{\partial D_{ij}} < 0 \).

**Proof of Proposition 4**

The characterization is given by the first order conditions for an interior policy.

**Proof of Proposition 5**

If \( D_{ii} = 1 \), then equations (5) and (7) are identical.

If \( D_{ii} = 0 \), then equation (5) becomes \( c_i(\hat{e}_i) - k_i(\hat{e}_i) = 0 \), while equation (7) becomes \( c_i(\hat{e}_i) - k_i(\hat{e}_i) = \delta \sum_{j \neq i} D_{ij} g(\hat{e}_j) > 0 \). We observe that the LHS of these two equalities are similar. Since \( c_i'(e_i) - k_i'(e_i) < 0 \), for these two equalities to hold, we must have \( \hat{e} \leq \hat{e} \).

Now let us examine the case where \( D_{ii} \in (0, 1) \). We argue by contradiction. Let us assume that there exists \( i \in I \) such that \( \hat{e}_i < \hat{e}_i \).
For the sake of simplicity we assume that the inequality is reversed for all other properties. Using a system of spatially differentiated fees enables us to implement the socially optimal policies under decentralized management. Using a rate on residual stock levels, the manager of property \( j \) faces an additional fee \( \tau_j e_j \) at each period \( t \) for a given residual stock level \( e_j \). According to our assumption, since the socially optimal residual stock level in property \( i \) is larger than that under decentralized management, the manager of this property should face a subsidy, that is, the optimal fee \( \tau_i \) should be negative (since the manager faces a minimization problem). It is easily checked that, under regulated decentralized management, one must have:

\[
 k_i(e_i) - c_i(e_i) + \tau_i + \delta D_{ii} c_i(x_i) g'(e_i) = 0.
\]

Assuming that the socially optimal policy is implemented, we obtain necessarily that (since we must have \( k_i(e_i) - c_i(e_i) = -\delta \sum_j D_{ij} c_j(x_j) g'(e_i) \)):

\[
 -\delta \sum_j D_{ij} c_j(x_j) g'(e_i) + \tau_i + \delta D_{ii} c_i(x_i) g'(e_i) = 0
\]

\[
 \Leftrightarrow \tau_i = \delta \sum_{j \neq i} D_{ij} c_j(x_j) g'(e_i)
\]

The right hand side of equality (34) is necessarily positive, which implies that \( \tau_i > 0 \) (the manager of property \( i \) faces a tax, not a subsidy), which is a contradiction.

**Proof of Proposition 6**

The result follows immediately from the first order conditions for a corner solution, \( e_i = 0 \), for any property \( i \).

**Proof of Proposition 7**

If complete eradication is a Nash equilibrium outcome, then for any property \( i \), due to Proposition 6 we have \( k_i(0) - [1 - \delta D_{ii} g'(0)] c_i(0) > 0 \). Since \( g'(0) \) is positive and all dispersal parameters are non negative, this implies that the following condition holds:

\[
 k_i(0) - [1 - \delta D_{ii} g'(0)] c_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij} c_j(0) \geq k_i(0) - (1 - \delta D_{ii} g'(0)) c_i(0) > 0.
\]

Using Proposition 6 enables us to conclude that complete eradication is socially optimal.

**Proof of Corollary 1**

The result follows immediately from conditions (8) and (9) in Proposition 6.

**Proof of Proposition 8**

The result follows immediately from conditions (8) and (9) in Proposition 6.

**Proof of Proposition 9**

From condition (14) we deduce that the length of the interval characterizing values of marginal abatement costs (in property \( i \)) over which tensions arise between socially optimal and decentralized management is given by

\[
 \Delta_i = \frac{k_i(0) + \delta g'(0) \sum_{j \neq i} D_{ij} c_j(0)}{1 - \delta D_{ii} g'(0)} - \frac{k_i(0)}{1 - \delta D_{ii} g'(0)} = \frac{\delta g'(0) \sum_{j \neq i} D_{ij} c_j(0)}{1 - \delta D_{ii} g'(0)}.
\]

This enables us to quickly deduce the following conclusions:
1. We have \( \frac{\partial \Delta_i}{\partial D_{ij}} = \frac{\delta g'(0)c_j(0)}{1 - \delta D_{ij}g'(0)} > 0 \) since \( 1 - \delta D_{ij}g'(0) > 0 \) and provided \( c_j(0) > 0 \), which implies that the length of the interval increases as dispersal increases. This concludes the proof of the first claim.

2. Again, differentiating with respect to the self retention rate, we obtain:

\[
\frac{\partial \Delta_i}{\partial D_{ii}} = \frac{(\delta g'(0))^2 \sum_{j \neq i} D_{ij}c_j(0)}{1 - \delta D_{ii}g'(0)} > 0
\]

since \( 1 - \delta D_{ii}g'(0) > 0 \) and provided \( \sum_{j \neq i} D_{ij}c_j(0) > 0 \), which implies that the length of the interval increases as self retention increases (provided that self retention remains lower than the threshold value \( \frac{1}{\delta g'(0)} \)). This concludes the proof of the second claim.

**Proof of Lemma 1**

Since condition (15) holds, we can deduce from the concavity of the payoff function that the manager of property \( j \) has incentives to increase the residual stock level compared to full harvest (provided that the manager of property \( i \) harvests the entire stock of the stock). This rules out complete eradication as a non cooperative equilibrium outcome. Now, if the manager of property \( j \) decides to increase the residual stock level (again assuming that the residual stock level is zero in property \( i \)) his optimal choice is given by \( \hat{e}_j \) (which is time and state independent). Then condition (16) enables to conclude that the manager of property \( i \) will find optimal to maintain the residual stock level in his property at zero (assuming that the manager of property \( j \) chooses \( \hat{e}_j \)). Thus at the Nash equilibrium the residual stock level will be zero in property \( i \) and positive in property \( j \). This concludes the proof.

**Proof of Proposition 10**

Let us compute that the gains from cooperation by the following expression:

\[
S = \hat{\Pi}_i + \hat{\Pi}_j - \hat{\Pi}_i - \hat{\Pi}_j = \frac{\delta}{1 - \delta} \left[ \int_0^{\hat{e}_j} c_i(s)ds + \int_0^{\hat{e}_j} k_j(s)ds + \int_{\hat{e}_j}^{D_{ij}g(\hat{e}_j)} c_j(s)ds \right] - \int_0^{\hat{e}_j} c_j(s)ds + \int_0^{\hat{e}_j} k_j(s)ds, \tag{35}
\]

which is positive by inspection. Moreover, it is easily checked that:

\[
\hat{\Pi}_i - \hat{\Pi}_i = \frac{\delta}{1 - \delta} \int_0^{D_{ij}g(\hat{e}_j)} c_i(s)ds > 0 \tag{37}
\]

\[
\hat{\Pi}_j - \hat{\Pi}_j = - \int_0^{\hat{e}_j} c_j(s)ds + \int_0^{\hat{e}_j} k_j(s)ds + \frac{\delta}{1 - \delta} \left[ \int_0^{\hat{e}_j} k_j(s)ds + \int_{\hat{e}_j}^{D_{ij}g(\hat{e}_j)} c_j(s)ds \right] \tag{38}
\]

Using expression (38) and the concavity of the payoff function, we deduce that

\[
\hat{\Pi}_j - \hat{\Pi}_j < \hat{e}_j \left[ k_j(\hat{e}_j) - c_j(\hat{e}_j) + \delta D_{ij}g'(\hat{e}_j)c_j(D_{ij}g(\hat{e}_j)) \right]. \tag{39}
\]

The right hand side of inequality (39) is equal to zero by condition (17), which enables us to conclude that the difference on the left hand side is negative. Thus, agent \( i \) gains from cooperation, while agent \( j \) would lose from it. Since gains from cooperation are positive overall, this implies that agent \( i \) would be willing to compensate agent \( j \), so that he would lower his residual stock level. This concludes the proof of the first statement of the proposition.
Proof of Proposition 11

We differentiate first the expression (35) with respect to parameter $D_{21}$, we obtain (keeping in mind that $\frac{\partial \hat{e}_j}{\partial D_{21}} = 0$):

$$\frac{\partial S}{\partial D_{ji}} = \frac{\delta}{1-\delta} g(\hat{e}_j)c_i (D_{ji}g(\hat{e}_j)) > 0,$$

which concludes the proof of the first statement of the proposition.

Second, differentiating with respect to $D_{jj}$, we obtain:

$$\frac{\partial S}{\partial D_{jj}} = \frac{\delta}{1-\delta} \left[ D_{ji}g'(\hat{e}_j)c_i (D_{ji}g(\hat{e}_j)) \frac{\partial \hat{e}_j}{\partial D_{jj}} + g(\hat{e}_j)c_j (D_{jj}g(\hat{e}_j)) \right],$$

(40)

with

$$\frac{\partial \hat{e}_j}{\partial D_{jj}} = -\frac{\delta g'(\hat{e}_j)}{SOC_j} \left[ g'(\hat{e}_j)D_{jj}c'_j (D_{jj}g(\hat{e}_j)) + c_j (D_{jj}g(\hat{e}_j)) \right].$$

using Proposition 3, we know that $\frac{\partial \hat{e}_j}{\partial D_{jj}} > 0$ if and only if the marginal cost is quite elastic. This concludes the proof.