Problem 1: (20 points) Given the matrices,
\[ A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & -2 & 3 \\ 0 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 1 \\ -1 & -2 \\ 3 \end{pmatrix} \]
calculate each of the following, or explain why it cannot be calculated.

(a) \( CB \)
(b) \( AB \)
(c) \( |A| \)
(d) \( C^T \)

Problem 2: (15 points) True/False. If the statement is always true, mark true. Otherwise, mark false. You do not need to show your work. (Any work will not be graded.)

(a) If \( x(t) \) is the solution to the equation of an overdamped spring, then there does not exist a value \( t_1 \) such that \( x(t_1) = 0 \).
(b) If a set of vectors \( W = \{w_1, w_2, \ldots, w_n\} \) is linearly independent in a vector space \( V \), then any subset of \( W \) is also linearly independent in \( V \).
(c) Let \( A \) and \( B \) be invertible \( n \times n \) matrices. Then \( |(AB)^{-1}| = \frac{1}{|A||B|} \).
(d) If \( \vec{x} = 0 \) is a solution to \( A\vec{x} = 0 \), then \( |A| \neq 0 \).
(e) Let \( \{v_1, \ldots, v_n\} \) be a basis for a vector space \( V \). Then for any \( y \in V, \{v_1, \ldots, v_n, y\} \) is linearly dependent.

Problem 3: (25 points) Consider the following system,
\[ \begin{align*}
x_1 + 4x_2 - 5x_3 &= 0 \\
2x_1 - x_2 + 8x_3 &= 9 \\
x_1 - 5x_2 + 13x_3 &= 9.
\end{align*} \]

(a) The system can be written as a matrix equation \( A\vec{x} = \vec{b} \). What are \( A, \vec{x} \) and \( \vec{b} \)?
(b) Reduce the associated augmented matrix to reduced row-echelon form.
(c) Find \( \vec{x}_h \), the solution of the homogeneous equation \( A\vec{x} = \vec{0} \).
(d) Find \( \vec{x}_p \), a particular solution of \( A\vec{x} = \vec{b} \).
(e) Use the nonhomogeneous principle to write down the general solution of the system of equations.
Problem 4: (20 points) Determine whether or not each of the following is a vector space. If it is a vector space, prove it. If not, show why not. **Hint:** Use the subspace theorem. You may assume that \( \mathbb{R}^n, \mathbb{P}_n, \mathbb{M}_{mn} \) and \( C(I) \) are all vector spaces.

(a) \( V = \) the set of all points in \( \mathbb{R}^2 \) that are on the line \( y = x \).
(b) \( V = \) the set of all invertible \( 2 \times 2 \) matrices.
(c) \( V = \) the set of continuous functions, \( f \) on \([0, 1]\) such that \( f(1) = 0 \).
(d) \( V = \{ (x, y, z) \mid x^2 + y^2 + z^2 \leq 1 \} \)

Problem 5: (20 points) An unforced mass-spring system is modeled by the equation
\[
m\ddot{x} + b\dot{x} + kx = 0.
\]

(a) A certain system has parameters \( m = 1, b = 5, k = 6 \). Find a general formula for the motion of the mass on the spring, for this system.
(b) A *particular* motion of this mass-spring system is obtained by stretching the spring a distance of one in the positive \( x \) direction and then releasing the mass from a standstill. Find the equation of motion—i.e., the solution to the IVP—in this instance.
(c) What is the long-term behavior of the solution in (b)? Does the solution ever reach zero?