1. (8 points each) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.
   
   (a) \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n} \]
   
   (b) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2} \]

   (c) \[ \sum_{n=1}^{\infty} \frac{\sin \left( \frac{2n-1}{2} \pi \right)}{n} \]

2. Consider the series \[ \sum_{n=0}^{\infty} \frac{(-1)^{n+1} (x - 1)^{n+1}}{n + 1} \].

   (a) (7 points) Find the radius of convergence.

   (b) (9 points) Find the interval of convergence.

3. The following problems are not related.

   (a) Consider the series

   \[ \left( \frac{1}{1 + 2!} \right) - \left( \frac{1}{2!} + \frac{2^3}{3!} \right) + \left( \frac{1}{4!} + \frac{2^5}{5!} \right) - \left( \frac{1}{6!} + \frac{2^7}{7!} \right) + \cdots = \sum_{n=0}^{\infty} \left[ (-1)^n \left( \frac{1}{(2n)!} + \frac{2^{2n+1}}{(2n + 1)!} \right) \right]. \]

   i. (7 points) Suppose we approximate the sum of the series by adding the first four terms (consisting of eight fractions). Estimate the error in the approximation.

   ii. (6 points) Find the exact sum of the series. (Hint: Refer to the formulas provided.)

   (b) Suppose \( f(x) \) equals its Taylor series \[ \sum_{n=0}^{\infty} \frac{(x + 1)^n}{e \cdot n!}. \]

   i. (6 points) Differentiate the series to find a power series representation for \( f'(x) \).

   ii. (6 points) Show that the series for \( f(x) \) and \( f'(x) \) are equal.

   iii. (7 points) The function \( f \) equals what commonly used function? Justify your answer using the definition of a Taylor series.
4. The following problems are not related.

(a) (7 points) The Maclaurin series for \( \frac{1}{\sqrt{5 + x}} \) is \( \frac{1}{\sqrt{5}} + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot \cdots \cdot (4n + b)}{\sqrt{5} n! c^n} x^n \). Find the values of the constants \( a, b, \) and \( c \).

(b) (9 points) Suppose \( f(x) \) has a Taylor series representation centered at \( a = 1 \) with radius of convergence \( R = 1 \) and \( f^{(n)}(x) = \frac{(-1)^n (n - 2)!}{x^{n-1}} \) for \( n = 2, 3, 4, \ldots \). Use Taylor’s formula to find an error bound if we approximate \( f(0.5) \) using \( T_4(x) \), the 4th order Taylor polynomial.

(c) (3 points each) Match the graphs shown below to the following parametric equations. Copy each graph into your bluebook and label it with the matching letter (a, b, c, or d). Then draw a single arrow on each graph to indicate the direction in which the curve is traversed. No explanation is required.

\[ \begin{align*}
(a) & \quad x = t + 1 \quad y = 2\sqrt{t} + 1 \quad 0 \leq t \leq 4 \\
(b) & \quad x = e^{-t} + 2t \quad y = e^t - 2t \quad -2.5 \leq t \leq 2.5 \\
(c) & \quad x = 2 \sin t \quad y = t + 1 \quad -6 \leq t \leq 4 \\
(d) & \quad x = 2 \cos t \quad y = t \quad -5 \leq t \leq 5
\end{align*} \]

Frequently Used Maclaurin Series:

\[ \begin{align*}
\frac{1}{1 - x} &= \sum_{n=0}^{\infty} x^n & R = 1 \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} & R = \infty \\
\tan^{-1} x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n + 1} & R = 1 \\
\ln(1 + x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} & R = 1 \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n + 1)!} & R = \infty \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} & R = \infty \\
(1 + x)^k &= \sum_{n=0}^{\infty} \binom{k}{n} x^n & R = 1
\end{align*} \]