1. (14 points) Consider the function \( f(x) = \cos^2 x \) on the interval \([-\pi, \pi]\).
   (a) On which intervals is \( f(x) \) increasing and on which intervals is \( f(x) \) decreasing?
   (b) Name any points of inflection. Make sure to verify the points are indeed points of inflection.

2. (16 points) For two resistors, \( R_1 \) and \( R_2 \), connected in parallel, the combined electrical resistance \( R \) is given by \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \). Further note that \( R \), \( R_1 \), and \( R_2 \) are all functions of time and are measured in ohms. \( R_1 \) and \( R_2 \) are each increasing at rates of \( \frac{1}{2} \) ohms per second. At what rate is the combined resistance changing when \( R_1 = 2 \) ohms and \( R_2 = 4 \) ohms?

3. Consider the function \( h(x) = \sqrt{25 - x^2} \).
   (a) (12 points) What is an appropriate linear approximation that could be used to estimate \( h(2.9) \)?
   (b) (2 points) Would the linear approximation of \( h(2.9) \) provide an overestimate or an underestimate?

4. (16 points) Consider the function \( f(x) = \sqrt{x^2 - 25} \).
   (a) Determine whether Rolle’s theorem can be applied to \( f(x) \) on \([-13, 13]\).
      If so, then find all values of \( c \) in \((-13, 13)\) satisfying the conclusion of the theorem.
      If not, then explain why the theorem does not apply in this instance.
   (b) Determine whether the Mean Value Theorem can be applied to \( f(x) \) on \([5, 13]\) .
      If so, then find all \( c \) guaranteed by the theorem.
      If not, then explain why the theorem does not apply in this instance.

5. (12 points) Determine the point(s) at which the graph of \( y^4 = y^2 - x^2 \) has a horizontal tangent. Hint: there is no horizontal tangent at the origin.

6. (28 points) Indicate, in your blue book, the following statements as True or False. No explanation required.
   (a) A point \( c \) in \((-2, 2)\) is guaranteed to exist such that the instantaneous rate of change of \( f(x) = \frac{x}{(x^2 + 1)^2} \) is \( \frac{1}{25} \).
   (b) \( g(x) = x^\frac{2}{3}(2 - x) \) has a local minimum at \( x = 0 \).
   (c) \( x = 0 \) is a critical point and local extremum of \( f(x) = x^4 - 2x^3 \).
   (d) The origin is a point of inflection for the function \( f(x) = x^6 \).
   (e) The function \( g(x) = \begin{cases} x^2, & -1 \leq x < 2 \\ 9 - 3x, & 2 \leq x \leq 3 \end{cases} \) has an absolute maximum of 4.
   (f) Every point where a function possesses a horizontal tangent is a local extremum of the function.
   (g) The point of inflection of \( f(x) = x(x - 6)^2 \) lies midway between the relative extrema of \( f \).

END of Exam