1. (28 pts, 7pts each) Evaluate each of the following.
(a) \( \int_{0}^{2\pi/3} (\sin \theta \cos^2 \theta + \sin^3 \theta) \, d\theta \)
(b) \( \int (2y - 1)^{3/2} - 1 \sqrt{2y - 1} \, dy \)
(c) \( \int_{e}^{5} \frac{1}{x \ln x} \, dx \)
(d) Use logarithmic differentiation to find \( \frac{dy}{dx} \) for \( y = \frac{\ln x}{(x + 1)(x^2 + 2)} \). Leave your answer unsimplified.

2. (15 pts) Using right hand endpoints, a definite integral is approximated by the Riemann sum:
\[
\sum_{i=1}^{n} \left[ \left( \frac{4i}{n} \right)^2 - 3 \right] \frac{4}{n}.
\]
(a) Find a definite integral represented by this Riemann sum.
(b) Evaluate \( \sum_{i=1}^{n} \left[ \left( \frac{4i}{n} \right)^2 - 3 \right] \frac{4}{n} \), that is, find the sum in terms of \( n \). Simplify your answer.
(c) Use either part (a) or part (b) to find the value of \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \left( \frac{4i}{n} \right)^2 - 3 \right] \frac{4}{n} \).
3. The following problems are not related.

(a) (5 pts) Write the sum in sigma notation. (Note: Do not try to find the value.)
\[ \frac{3}{7} - \frac{4}{8} + \frac{5}{9} - \frac{6}{10} + \cdots + \frac{23}{27}. \]

(b) (7 pts) Water is flowing into a tub at \(3t + \frac{1}{t+1}\) gallons per minute. Assuming the tub started with 10 gallons of water at time \(t = 0\), how much water is in the tub after 2 minutes?

(c) (7 pts) Use Newton’s Method to find a root of the equation \(x^3 - 7x - 6 = 0\). Start with an initial guess of \(x_1 = 1\) and find \(x_2\) and \(x_{100}\).

(d) (7 pts) Let \(f(x) = \int_2^x \sqrt{1 + t^3} \, dt\). Show that \(f\) is one-to-one (i.e. so it has an inverse) and find \((f^{-1})'(0)\).

(e) (7 pts) Find the average value of the function \(f(x) = x(\sqrt[3]{x} + \sqrt[5]{x})\) on \([-1, 1]\).

4. (24 pts, 4 pts each) Consider the function \(y = \sin(t^2)\), shown below.

Let \(g(x) = \int_{-a}^x \sin(t^2) \, dt\), \(-a \leq x \leq f\). Answer the following questions about \(g(x)\). Your answers to parts (iii), (iv), and (v) will be in terms of \(a, b, c, d, e,\) and \(f\). No justification is needed for this problem.

(i) Find \(g'(x)\).

(ii) Find \(g''(x)\).

(iii) On which interval(s) is \(g\) decreasing?

(iv) At what value(s) of \(x\) does \(g\) have local minimum values?

(v) Suppose we wish to estimate the value of \(g(f)\). Calculate the lower and upper sums using \(n = 1\) subinterval.

(vi) Now find the numerical value of \(a\) and use it to find the numerical value of \(g''(a)\).

Formulas
\[
\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2
\]