The Transformation of American Political Space 1982-2002

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The range of acceptable positions about important political issues defines the political space of society. This paper presents a model of how political space is transformed over time based upon linear differential equations. Constructable transformations of political space are ones that can be modeled by such equations and thus can result from gradual evolutionary change. Non-constructable transformations cannot be represented in this way because they embody political discontinuities of some kind. A simple typology of constructable transformations is discussed and illustrated. Methods of estimating the political space transformation model are outlined and applied to General Social Survey data for 1982, 1993, and 2002. This leads to an analysis of how American political space transformed from 1982 to 1993 and then from 1993 to 2002. The former transformation proves to be constructable while the latter is not, probably due to the effects of 9/11. The 1982 to 1993 period witnessed a tentative simplification of American political space featuring some left-right polarization. The 1993 to 2002 period is characterized by a conservatizing transformation of American political space which, the analysis suggests, could result from the attacks of 9/11.
1. *Introduction*

The political climate of society sometimes changes dramatically. For example, the political climate of American society underwent formidable change after September 11, 2001 (Mann 2003). Change of political climate is usually understood as the result of shifts in the political ideas or attitudes of citizens. Surely this is an important part of the process, but it may not be the entire story. Human beings influence each other, and when many people change their political ideas it sets up a complex field of social forces. Persons experience their own political changes as at least partly a response to an external social reality (Gamson 1992, Benford and Snow 2000). When this happens, greater insight and clarity can result from conceiving the political climate of society, not as an aggregate of individual positions, but as an objective field or space. Persons occupy locations within that political space. Of course political actors are not mindless robots, but they do locate themselves within a political space the overall structure of which they cannot control.

The concept of political space is particularly useful for understanding major changes in political climate (Sack 1980, Schatski 1991). The magnitude, ubiquity, and simultaneity of such changes suggests that they are better analyzed as transformations of the political space rather than as aggregations of individual change. To facilitate such analysis, we shall separate transformations of political space into systematic and contingent components and focus upon the former. Systematic transformations are conceived as ones that proceed in an orderly manner over a relatively long period of time. Within the realm of systematic transformations of political space, we make a further distinction between *constructable* and *non-constructable* transformations. A constructable transformation is a change in political space that could arise through certain simple but plausible processes. The word “could” is essential here. We do not claim that a constructable transformation
actually did result from these simple processes, but only that it could have done so. On the other hand, a non-constructable transformation of political space is an orderly change that could not happen in this way.

The difference between constructable and non-constructable transformations of political space has important evolutionary meaning. A constructable transformation of political space can result entirely from gradual change, and can be decomposed into arbitrarily small and homogeneous steps. By contrast, a non-constructable transformation, though unfolding in an orderly manner and hence systematic, cannot be the cumulative outcome of many tiny and essentially similar changes. The process of change must include a qualitative leap of some kind. We demonstrate below that constructable transformations of political space have five elementary or canonical forms, each of which describes a different kind of political change. Observed constructable transformations typically combine these elementary forms in complicated ways, but the five-fold classification illuminates the basic dynamics or building blocks of evolutionary political change (Hasselblatt and Katok 2003).

This paper proposes a new way of thinking about political change and applies it to the United States between 1982 and 2002. The first half of the paper explains the underlying concepts, provides simple examples that clarify their meaning, and develops the technical apparatus needed to apply these ideas. The second half of the paper proposes estimation methods and analyzes the transformation of American political space over two recent decades. Among other things it offers an estimate, or rather a projection, of the long term political effects of 9/11. Although the empirical analysis is elaborate – and I think informative – it must be considered exploratory because it uses information gathered for other purposes and estimation procedures still under development.
The next section discusses the concept of political space. The two subsequent sections explain the meaning of constructable transformations of political space and provide elementary examples. Section five develops some concepts useful for analyzing transformations, and also presents a necessary condition for constructability. The next section identifies the five canonical transformations of political space, discusses their distinctive properties, and offers simple examples of how they work. Section seven outlines the procedures used to estimate transformations of political space, after which these procedures are applied to the United States in two stages. Section eight analyzes the transformation of American political space between 1982 and 1993, a transformation that turns out to be constructable. Section nine proposes methods for dealing with non-constructability and uses these methods to examine the non-constructable transformation of American political space between 1993 and 2002.

This paper employs mathematical reasoning to analyze transformations of political space. However, the basic ideas do not require mathematics. They arise from sociological theory and informal observation of political change. A substantial amount of mathematical symbolism is unavoidable. Mathematical ideas are needed to identify constructable transformations, distinguish the five canonical forms, estimate transformations of political space from empirical data, and cope with non-constructability. Yet I have also tried to accommodate the non-mathematical reader. The intuitive and practical meanings of important mathematical concepts are carefully explained and illustrated. If one can tolerate certain leaps of faith, mathematical symbols can be skipped without losing the essential thread of the argument.
2. The Concept of Political Space

A political system contains a finite number of durable general issues on each of which a range of locations is possible. These durable general issues may be called political dimensions. A political position is defined by a complete set of locations on these political dimensions. A political space is the full range of plausible (i.e. defensible, or acceptable, or legitimate) political positions available within a political system. The crucial feature of a political space is the interactive relationship with the people who occupy positions within it. The space emerges from the totality of political positions, but it also exists independently of them. This relative autonomy of position from occupant allows us to analyze transformations of political space without making claims about how individual people will change political positions.

The concept of political space has much in common with the concept of field as used by thinkers like Kurt Lewin (1951) and Pierre Bourdieu (1985, 1988, 1989). In his recent review of field theory, John Levi Martin says:

“field theory is an excellent vehicle for making complex social phenomena intuitively accessible....field theory elegantly handles as fundamentally the same two social phenomena usually considered to be antithetical, namely the feeling that there is some social force that constrains individuals externally, and the feeling that we act on the basis of our motivations.”

(2003, pp. 36-7)

The concept of political space has many of these same virtues. By studying changes in political space, we examine the simultaneous transformation of social forces that constrain individuals.

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1 Informative and surprisingly relevant discussions of how the space concept is used in the physical sciences and mathematics appear in Reichenbach (1958), Jammer (1993), and Greene (2003).
A metric is a function $D(x,y)$ that specifies a distance between any two points in space (x and y). A metric has three essential characteristics: (1) [positivity] $D(x,y) \geq 0$ and $D(x,y) = 0$ if and only if $x = y$; (2) [symmetry] $D(x,y) = D(y,x)$; (3) [triangle inequality] $D(x,y) + D(y,z) \geq D(x,z)$. The standard Euclidean metric for n-dimensional space is defined as $D(x,y) = [(x_1 - y_1)^2 + \cdots + (x_n - y_n)^2]^{1/2}$.

Numerical representation makes it possible to define a metric on the political space. This enables measurement of the distance between political positions and determination of whether a particular transformation alters or preserves this distance. It also allows analysis of how transformations effect entire regions of political space. Does the transformation expand or contract the volume of a particular region? Does it change the shape of the region in some special way? Although many different metrics can be defined on the political space, we shall generally use the

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2 A metric is a function $p(x,y)$ that specifies a distance between any two points in space (x and y). A metric has three essential characteristics: (1) [positivity] $p(x,y) \geq 0$ and $p(x,y) = 0$ if and only if $x = y$; (2) [symmetry] $p(x,y) = p(y,x)$; (3) [triangle inequality] $p(x,y) + p(y,z) \geq p(x,z)$. The standard Euclidean metric for n-dimensional space is defined as $p(x,y) = [(x_1 - y_1)^2 + \cdots + (x_n - y_n)^2]^{1/2}$. 

The major dimensions of politics vary from one political system to another because the general issues important within these systems differ (Gregg and Banks 1965, Sharansky and Hofferbert 1969, Paine 1989). The political space of the United States, discussed in the second half of this paper, is analyzed as having seven political dimensions defined as ideological locations regarding class, race, gender, civil liberties, crime, military affairs, and the environment. Other research I am doing on German fascism suggests seven somewhat different political dimensions: capitalism, authoritarianism, nationalism, racism, violence, militarism, and conservatism. A suitable political dimension is continuous and one dimensional. It amounts to a one dimensional scale and can be represented by a real number interval. Complex political ideas must be decomposed into several different dimensions. Thus a location on a political dimension is indicated by a single real number, while a political position is represented by a vector of real numbers one for each political dimension.
The dimensions of a political space can be identified by observing the actual practice of the political system. Although the number of different topics that arise within a modern political system tends to be quite large, the set of durable general issues is much smaller and through suitable abstraction can usually be reduced to a manageable size. In keeping with both conventional left-right terminology about politics and standard graphical representation, wherever possible negative numbers are used to code left wing or liberal political locations and positive numbers are used to code right wing or conservative locations. The initial midpoint of the political spectrum is coded zero. In the graphs presented below, upward movement indicates an increasingly conservative trajectory, while downward movement signifies an increasingly liberal path. These scaling conventions should make both numerical and geometric representations more intuitively accessible, but do not change the deeper analysis.

Political space is not static. It undergoes both abrupt shocks and systematic transformations. The focus of this paper is on the latter, but we will analyze at least one important political shock. The pace of systematic transformation can be slow and almost undetectable, or speedy and breathtaking, or anything between these extremes. Different political dimensions usually transform at different rates. When a political space is systematically transformed, each point or position in the space maintains its integrity and is mapped into a distinct new position. This one-to-one mapping is a crucial assumption of the transformation model proposed herein. One-to-one mapping does not preclude expansions, contractions, or radical distortions of the political space (Arnold 1993, Katok and Hasselblatt 1995).
movements of persons and positions as theoretically separable.

The entire conception of space transformation is a structuralist one (Burt 1992; White, Boorman, and Breiger 1976). The social forces that reshape political space simultaneously move or map the old location into the new one. Here the distinction between individuals on the one hand and institutions or collectivities on the other becomes relevant. Whereas individual change need not follow the structural movement of political positions, we shall argue that collective or institutional movement does tend to do so. This happens because collectivities and institutions are complex compositions of many individuals and thus less subject to particular idiosyncracies (Hedström and Swedberg 1998). The ideological and strategic movements of political parties, corporations, labor unions, government agencies, newspapers, churches, and advocacy organizations do manifest the transformation of political space. Indeed, we shall use such collective change to identify how political space is transformed within observable political systems (Manza and Brooks 1997).

3. The Nature of Constructable Transformations

As stated above, a constructable transformation is a systematic change of political space that could result from certain simple but plausible processes. How does a constructable transformation of political space work? It has several distinctive characteristics. A constructable transformation unfolds steadily over a period of time. The transformation is smooth and, when considered over a short time period, roughly proportional in magnitude to the length of the period. Hence the amount of political change becomes arbitrarily small when considering a sufficiently short time interval. A constructable transformation of political space results from social forces that operate consistently over time. Although the general principle of transformation is uniform, the specific impact of these forces depends upon the particular location within political space being transformed. Thus it is
possible to associate a vector of change with each location in the space, and these change vectors are themselves continuous. We call a transformation that satisfies all these conditions *constructable*.

A constructable transformation, as it unfolds over time, is a continuous process of change. Because it is propelled by forces that operate in a consistent manner and whose impact depends upon location within political space, it is natural to model a constructable transformation by system of ordinary differential equations. The simplest class of ordinary differential equations with sufficiently flexibility to represent a broad variety of constructable transformations are linear systems (Arnold 1992, chapter 3; Perko 1996, chapter 1). Indeed the role of linear systems within the larger domain of differential equations is comparable to the role of linear multiple regression within the larger domain of regression models.4

Systems of differential equations model how each point (i.e. political position) of the political space is transformed. One equation corresponds to each dimension, and thus the number of equations in the system equals the number of dimensions in the space. Each differential equation depicts how a political dimension changes. If the system of equations is linear, then the rate at which location on any specific dimension changes is a linear function of the overall position within political space. Thus the rate at which a specific dimension changes depends upon the entire political position not just that particular dimension.

Suppose that the political space have n dimensions, and let vector \( X(t) = [x_1(t), x_2(t), \ldots, x_n(t)] \) be the coordinates of a point at time \( t \). The system of n linear differential equations representing space transformation would then be written as

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4 More generally, it can be shown that any group of linear transformations indexed by time can be represented in this manner (Arnold 1992, pp. 165 and 182).
One of the virtues of linear systems of differential equations is that they can be used to approximate non-linear systems within limited ranges. For a sociological application of non-linear differential equation systems see Mayer (2002).

\[
\frac{dx_1(t)}{dt} = a_{11}x_1(t) + a_{12}x_2(t) + \cdots + a_{1n}x_n(t) + b_1 \\
\frac{dx_2(t)}{dt} = a_{21}x_1(t) + a_{22}x_2(t) + \cdots + a_{2n}x_n(t) + b_2 \\
\vdots \\
\frac{dx_n(t)}{dt} = a_{n1}x_1(t) + a_{n2}x_2(t) + \cdots + a_{nn}x_n(t) + b_n
\]

(1)

where the \(a_{ij}\) and \(b_i\) terms are constant parameters. The parameter \(a_{ij}\) specifies how much each unit of coordinate \(x_j(t)\) contributes to the rate of change of coordinate \(x_i(t)\). The parameter \(b_i\) indicates a constant contribution to the rate of change of coordinate \(x_i(t)\). If all the \(b_i\) terms equal zero, then the set of equations above is a homogeneous system. If not all \(b_i\) terms equal zero then the system is inhomogeneous. For purposes of simplicity and compactness, the system of equations in (1) is usually written in matrix notation as follows

\[
X = \begin{bmatrix}
x_1(t) \\
\vdots \\
x_n(t)
\end{bmatrix}, \quad \frac{dX(t)}{dt} = \dot{X} = \begin{bmatrix}
\frac{dx_1(t)}{dt} \\
\vdots \\
\frac{dx_n(t)}{dt}
\end{bmatrix}
\]

(2)

\[
A = \begin{bmatrix}
a_{11} & \cdots & a_{1n} \\
\vdots & \ddots & \vdots \\
a_{n1} & \cdots & a_{nn}
\end{bmatrix}, \quad B = \begin{bmatrix}
b_1 \\
\vdots \\
b_n
\end{bmatrix}
\]

\[
\dot{X} = AX + B
\]

The square matrix \(A\) is sometimes called the infinitesimal generator of the linear system, and we shall use this terminology.

If linear systems of differential equations are indeed appropriate models for transformations of political space, then the general properties of these systems should reveal something about the general nature of political transformations. As we shall see, linear systems exhibit a number of qualitatively different patterns, which also occur in the constructable political transformations found in the real world. A simple typology of these qualitative patterns, with reference to constructable
transformations of political space, is presented in section six. Before this typology will make sense, however, some additional information about linear systems of differential equations is required.

The basic differential equations that define a linear system involve instantaneous rates of change that are not directly observable. To make these equations empirically meaningful their observable implications must be determined. This involves solving linear systems (1) and (2) and ascertaining the value of the political position vector \( X(t) \) for any time \( t \) greater than zero. The solution, it turns out, depends upon the initial political position \( X(0) \) as well as of matrix \( A \) and vector \( B \). Indeed, the function that transforms \( X(0) \) into \( X(t) \) is exactly the constructable transformation of political space that we are trying to identify.

It is actually rather easy to write the solution of linear systems (1) and (2) using the concept of the matrix exponential. This is a generalization of the exponential function \( e^t \) from scalar numbers to matrices, and is symbolized as \( e^{At} \) where \( A \) is a square matrix and \( t \) is a scalar number (in our context always representing time). In fact the matrix exponential equals (by definition) the same power series as the exponential function

\[
e^{At} \equiv \sum_{k=0}^{\infty} \frac{A^k t^k}{k!}.
\]

The solution to the homogeneous system is particularly simple to write. If

\[
\dot{X}(t) = AX(t)
\]

then

\[
X(t) = e^{At}X(0), \quad t \geq 0. \tag{5}
\]

\footnote{This result is sometimes referred to as the fundamental theorem of linear differential equations with constant coefficients (Arnold 1992, p. 164).}
4. Elementary Examples

We will make extensive use of the matrix exponential in studying how political space is transformed. As a preliminary step consider three elementary examples involving political spaces with only two dimensions. All three examples have been simplified as much as possible to illustrate the basic underlying processes. To be concrete, suppose that the first or horizontal dimension measures domestic policy ideology, while the second or vertical dimension measures ideology about foreign policy. The three examples along with a translation of political space are diagramed in Figure 1. [[Insert Figure 1 about here.]]

First consider a transformation in which each dimension impacts only itself. In this case the infinitesimal generator $A$ is a diagonal matrix. For example

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad e^{At} = \begin{bmatrix} e^{2t} & 0 \\ 0 & e^t \end{bmatrix}.$$  

This corresponds to a transformation that simply stretches the two political ideologies by factors of $e^{2t}$ and $e^t$ respectively leaving their centers (zero values) unchanged. Both left and right ideologies become more extreme with time, and domestic policy location (with expansion factor $e^{2t}$) changes more than foreign policy location (with expansion factor $e^t$). A transformation of this kind is usually called a dilation and could involve any combination of political stretching and political contraction (see diagram one of Figure 1).

Next consider a very different transformation of our two dimensional political space in which each dimension impacts the other but not itself. That is, domestic policy ideology influences how foreign policy ideology changes (and vice versa), but neither one influences its own rate of change. Moreover, assume that domestic policy and foreign policy ideologies have exactly opposite impacts
A skew-symmetric matrix has the property that the transpose of the matrix equals the negative of the matrix: $A^T = -A$. An orthogonal matrix is a square matrix whose transpose equals its inverse: $B^T = B^{-1}$. If $A$ is a skew-symmetric matrix, then $e^{At}$ will be an orthogonal matrix (Gantmacher 1959, p. 287).

(7) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $e^{At} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

Here the transformation is a rotation of political space counter-clockwise through an angle of $t$ radians around the political center (see diagram two of Figure 1).

In such a rotation, all positions in political space, regardless of how much they change, remain exactly the same distance apart. This certainly does not happen in the political dilation considered previously. Here, however, the rotated ideologies make compensating changes. For example, when domestic policy moves further away from zero (its center location) then foreign policy moves closer to zero. As a consequence of such compensating changes, the distance of each position in the two dimensional political space from the overall political center (i.e. the origin of the space) does not change. Rotations are easy to visualize in two dimensional space, but they can also occur in spaces of larger dimensions. Rotations in n-dimensional space are called orthogonal transformations.

A third transformation involves a small but significant difference from the example just given. Assume once again that domestic policy and foreign policy ideologies impact each other but not themselves. In contrast to the previous example, however, assume that each ideology has the same rather than the opposite effect on how the other one changes. Matrix $A$ would then be symmetric as

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7 A skew-symmetric matrix has the property that the transpose of the matrix equals the negative of the matrix: $A^T = -A$. An orthogonal matrix is a square matrix whose transpose equals its inverse: $B^T = B^{-1}$. If $A$ is a skew-symmetric matrix, then $e^{At}$ will be an orthogonal matrix (Gantmacher 1959, p. 287).

8 A transformation which does not change the distance between points is called isometric.

9 Generically, that is with almost no exceptions, orthogonal transformations in higher dimensions can be reduced to a number of separable two dimensional rotations.
The functions \( \cosh t = \frac{1}{2}(e^t + e^{-t}) \) and \( \sinh t = \frac{1}{2}(e^t - e^{-t}) \) are the hyperbolic sine and cosine.

\[ A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } e^{At} = \begin{bmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{bmatrix} \]

This transformation, like the infinitesimal generator that produced it, is symmetric.\(^{10}\) It turns out to be a special kind of dilation. The dilation is not along the regular coordinate axes, but in directions exactly 45 degrees to them. In one of these directions, stretching by a factor of \( e^t \) occurs. In the other direction, which is orthogonal to the first, contraction by the inverse factor of \( e^{-t} \) happens. Thus positions where domestic policy and foreign policy ideologies have equal numerical measures move further away from the origin by a multiplier of \( e^t \), while positions where they have opposite measures move closer to the origin by a multiplier of \( e^{-t} \). This transformation provides an example of an asymmetric or distorting dilation (see diagram three of Figure 1). It changes the shape of politically defined regions by stretching them in one direction and contracting them in another. In fact it suggests that only political positions in which domestic policy and foreign policy ideologies are almost identically located on the liberal-conservative continuum remain legitimate.\(^{11}\) In contrast to the previous example, this symmetric transformation does not preserve the distance between political positions (is not isometric). We shall see, however, that it does preserve the volume of political space.

The discussion above concerns homogeneous transformations of political space, that is

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\(^{10}\) The functions \( \cosh t = \frac{1}{2}(e^t + e^{-t}) \) and \( \sinh t = \frac{1}{2}(e^t - e^{-t}) \) are the hyperbolic sine and cosine.

\(^{11}\) To see this, consider a circle centered on the origin. The transformation in (8) changes the circle into an ellipse of exactly the same area. The center of the ellipse remains at the origin, but its major axis lies on the 45° line, while its minor axis lies on the -45° line. As time passes the ellipse becomes progressively more eccentric and more concentrated around the 45° line, while always maintaining the same area. Eventually the circle is distorted into a long but exceedingly narrow ellipse the points of which become arbitrarily close to the 45° line. Along this line domestic policy and foreign policy ideologies have identical numerical values.
constructable transformations governed by homogeneous linear systems. All such transformations
leave the origin of the political space unchanged. Transformations governed by non-homogeneous
linear systems (i.e. containing a constant vector $B$), on the other hand, shift the origin of the space as
well as other positions. If the infinitesimal generator $A$ is non-singular (i.e. has an inverse matrix $A^{-1}$)
then the general solution of the non-homogeneous systems (1) and (2) is straight forward

$$X(t) = e^{At}X(0) + A^{-1}[e^{At} - I]B$$

Moreover, a fixed point of this non-homogeneous political transformation occurs at $-A^{-1}B$.

A mathematically trivial but conceptually significant political transformation happens when
every position in political space is subject to exactly the same forces. This means that position has
no influence on the transformation process, and hence the infinitesimal generator $A$ becomes the zero
matrix. Under these circumstances the transformation depends entirely upon initial position $X(0)$, the
constant vector $B$, and the length of time $t$. Then the solution is simply

$$X(t) = X(0) + Bt.$$  

This is called a translation of the political space. It involves a movement of the entire space in the
direction specified by vector $B$ and by an amount proportional to the elapsed time $t$ (see diagram four
of Figure 1). A translation leaves no position of the space unchanged, but preserves the distance
between all political positions (it is isometric).

5. Volume Change, Subspaces, and Constructability Conditions

The volume of political space can expand or contract. If the volume of political space
expands, more political positions become realistically available, and tightly clustered positions
separate from each other. A rapid expansion of political space could undermine existing political alliances and perhaps lead to the formation of new ones. A political space contraction has quite different effects. It simplifies political space either by contracting political positions towards an equilibrium state or by reducing the number of effective political dimensions. Movement towards an equilibrium tends to lessen political discord, but simplification in the form dimension reduction can have the opposite effect. It can increase political separation and polarize positions within a functionally reduced political space. As we shall see, something like the latter happened to the United States between 1982 and 2002.

Whether a constructable transformation is expanding or contracting (or neither) depends entirely upon its infinitesimal generator $A$ and the associated matrix exponential $e^{At}$. It can be shown that any linear system of the form given in (1) and (2) changes the volume of any region in space by a factor equal to the determinant of matrix $e^{At}$. It can also be shown that

$$\det e^{At} = e^{\text{trace } A}, \quad \text{where } \text{trace } A = \sum_{i=1}^{n} a_{ii}. \quad 12$$

Thus whether a constructable political transformation is space expanding, contracting, or preserving depends upon whether the sum of the infinitesimal generator’s diagonal elements (i.e. $\text{trace } A$) is positive, negative, or zero. Inspecting the three examples given above, we see that (6) is space expanding while (7) and (8) are space preserving.

Liouville’s formula (11) also implies that the determinant of $e^{At}$ is greater than zero. This has strong implications about the constructability of political space transformations. It implies that many conceivable transformations are not constructable. If a political transformation is not constructable,

\footnote{These results are known as Liouville’s formula (Arnold 1992, p. 172).}
then it cannot happen within the given political space through a sequence of small changes.\footnote{The argument above applies only to linear systems. But smooth non-linear systems are well approximated over short time intervals by linear systems, and hence the non-constructability result applies to any such systems as well.} Hence the transformation cannot result from a gradual evolution. For example, in the two dimensional political space defined by domestic and foreign policy ideologies, a transformation that maintains domestic policy [foreign policy] position but reverses foreign policy [domestic policy] position (multiplies this value by -1) is not constructable. Such a transformation amounts to a reflection of political space about the domestic policy [foreign policy] axis (see diagram 4 of Figure 2). [[Insert Figure 2 about here]]

Similarly, a transformation of political space in which domestic policy and foreign policy positions are switched (the value of the first becomes the value of the second and vice versa) is also not constructable. This is equivalent to a reflection of political space about the line bisecting the two axes (see diagram 3 of Figure 2). In both these cases the matrix of the transformation has a negative determinant and thus cannot be reproduced by any transformation of form $e^{\mu}$. This is surprising, because the more radical political transformation in which both domestic policy and foreign policy attitudes are reversed is indeed constructable. This simply involves 180 degree rotation of political space (see diagram 1 of Figure 2) and can be achieved with the infinitesimal generator in (7).

The implications of constructability in three dimensional political space are even more disconcerting. Transformations requiring reflections are not constructable.\footnote{Non-constructability applies only to transformations of the entire political space, not to individual political changes. A person can make virtually any political change in a gradual evolutionary manner, but non-constructable transformations by the entire political space always have some discontinuity.} Consider a three dimensional political space defined by ideologies about economic policy, civil liberties, and foreign policy. A transformation in which any one of these three ideologies is reversed while the other
ideologies remain fixed (i.e. reflection about the plane defined by the two unchanging ideologies) is not constructable. Neither is a transformation in which all three political ideologies are reversed (i.e. three reflections or one reflection and a rotation), nor one where two positions switch – say economic policy and foreign policy positions – while the third remains fixed (i.e. reflection in a diagonal plane). On the other hand, a political transformation in which any two ideologies are reversed while the third remains fixed (i.e. rotation around the axis of the fixed ideology) is constructable, as is a transformation involving a complete permutation of positions, e.g. economic policy → civil liberties → foreign policy → economic policy (i.e. rotation around the equal position vector).

These are not obscure mathematical facts. On the contrary, they reflect the difference between systemic political changes that can happen through arbitrarily small steps and systemic changes that require qualitative leaps. The examples above were chosen to foster intuition about the difference between constructable and non-constructable transformations. In practice, the issue of whether a given transformation of political space is constructable or not is rarely obvious. A necessary but not sufficient condition of constructability is that transformation preserve the orientation of the political space.\(^{15}\) Roughly speaking this means that a constructable transformation does not reflect or “flip over” the space. Flipping over an entire space is a qualitative operation that cannot be accomplished in small steps.

The political space of any complex modern society has many dimensions. Often it is not feasible to study all these dimensions simultaneously. However, if the political space contains an autonomous subspace, its transformation can be analyzed without considering the entire space. A political subspace is autonomous if the way its dimensions change depends only upon dimensions

\(^{15}\) A linear transformation is orientation preserving if its determinant is positive. Note that the determinant of the infinitesimal generator $A$ can be negative, but by Liouville’s formula (11) the determinant of $e^{At}$ is always positive.
within that subspace. Dimensions external to an autonomous subspace have no influence on how any dimension included therein changes. The order in which political dimensions are listed is arbitrary. Suppose that \( k \) dimensions of an \( n \)-dimensional political space constitute an autonomous subspace, and that the order of dimensions is rearranged to put these \( k \) dimensions first. The \( n \) by \( n \) infinitesimal generator can then be partitioned as follows

\[
A = \begin{bmatrix}
B_{k \times k} & O_{k \times n-k} \\
C_{(n-k) \times k} & D_{(n-k) \times (n-k)}
\end{bmatrix}
\]

where the subscripts indicate the dimensions of the four sub-matrices. The crucial feature of this partition is sub-matrix \( O_{k \times n-k} \), which is a zero matrix having \( k \) rows and \( n-k \) columns. These zeroes indicate that none of the remaining \( n-k \) dimensions impacts any of the first \( k \) dimensions. This feature allows the \( k \) by \( k \) sub-matrix \( B_{k \times k} \) to function as the infinitesimal generator of the \( k \)-dimensional autonomous subspace. The transformation results for this subspace are the same as if the entire \( n \)-dimensional space is analyzed.\(^{16}\)

When analyzing the transformation of United States political space below we consider only seven political dimensions. Although American political space could have many other political dimensions, we shall assume that these seven dimensions constitute an autonomous subspace.

6. **Canonical Transformations of Political Space**

The word “canonical” as used in this section means simple, clear, and basic. Canonical

\(^{16}\) If sub-matrix \( C_{n \times k} \) is also a zero matrix, then the second set of \( n-k \) dimensions is also an autonomous subspace with sub-matrix \( D_{k \times k} \) as its infinitesimal generator. If this happens the political space decomposes into a pair of autonomous subspaces. It is theoretically possible for a political space to decompose into even more than two autonomous subspaces.
transformations of political space are not those that appear most often. In fact they rarely occur in anything like a pure form. Canonical transformations are basic because their effects are conceptually simple, and because more complex transformations – if they are constructable – arise from compositions of these elementary ones. The meaning of a more complex transformation is clarified by decomposing it into a sequence of canonical transformations. Moreover, any combination of canonical transformations will be a constructable transformation.

The examples in the previous section introduce some of the five canonical transformations of political space. Canonical transformations are of two general types: distance preserving or isometric transformations, and distance altering or non-isometric transformations. As indicated above, the defining feature of a distance preserving transformation is that it fixes the distance between every pair of political positions. An isometric transformation changes political positions, but not the distances between these positions. It is a “rigid” transformation of political space. A distance preserving transformation is also volume preserving: the volume of any region in political space remains constant under the transformation. Two of the five canonical transformations are distance preserving: political translations and political rotations (e.g. example (7) above). Any combination of a political translation and a political rotation is also distance preserving and, conversely, every distance preserving transformation of political space is some combination of a translation and a rotation.

Distance altering canonical transformations are of three general types: political unifications or contractions, political polarizations or dispersions, and political reconfigurations or asymmetric dilations. In the literature on dynamical systems all three of these transformations are called dilations and analyzed by identical methods, but from the perspective of social science they differ significantly. While political translations and rotations can be combined, political unifications, dispersions, and
reconfigurations are mutually exclusive. However, each member of this trio can be amalgamated with translations and rotations. Indeed, all the historical transformations of political space I have examined involve just such combinations. Political unifications contract the volume of political space; political dispersions expand this volume; and political reconfigurations can be either volume expanding, volume contracting, or volume preserving.

The meaning of a political translation has already been discussed (see diagram four of Figure 1). It is a movement of the entire political space in a fixed direction by a fixed amount. The translation is fully described by the single vector that is added to each position in the political space. Not only is political translation distance preserving, it also preserves relative position on each political dimension. For example, if position $\alpha$ is 3 units more liberal on foreign policy than position $\beta$ prior to the translation, then it remains 3 units more liberal after the translation. A political translation is of course a non-homogeneous transformation because it changes the origin in the same way that it changes every other position in the political space. Any constructable political transformation that moves the origin includes a political translation. The transformation of the origin provides a means of estimating the political translation component of the overall transformation.

The second distance preserving transformation is the political rotation. A two dimensional example of this was given in (7) above. A political rotation entails compensating changes in different political ideologies so that overall distance from the political center remains unchanged. More precisely, a political rotation has two essential properties: (a) it preserves the distance between all political positions, and (b) it leaves the political origin fixed (and thus differs from a political translation). Within political spaces of two or three dimensions, these properties correspond to the usual understanding of rotation. In two dimensional space the only transformations that satisfy
properties (a) and (b) are simple rotations of the plane around the origin. And in three dimensional space, only rotations around a fixed axis intersecting the origin meet these two requirements (Artin 1991, pp. 123-130).

With more than three dimensions the meaning of a political rotation is difficult to visualize. It turns out that requirements (a) and (b) are satisfied if and only if the matrix exponential $e^{At}$ is an orthogonal matrix and the vector of constants $B$ in (2) is zero. As indicated in footnote 7, a necessary and sufficient condition for the orthogonality of $e^{At}$ is that the infinitesimal generator $A$ be a skew symmetric matrix.$^{17}$ Actually the meaning of rotations in higher dimensional space is not quite so obscure. It still rests upon the notion of compensating changes between two political dimensions. Dimensions can almost always be redefined in such a way that a higher dimensional rotation becomes several independent two dimensional rotations. Thus a higher dimensional rotation is equivalent to several independent compensating changes among pairs of suitably defined political dimensions (e.g. rotations in three dimensional space mentioned above).

A political rotation is not simply a mathematical fantasy. The argument above shows that rotation, even in multi-dimensional political space, is essentially a two dimensional phenomenon. Its practical importance arises from two fundamental political processes: balancing and identity maintenance. The balancing process entails compensating or balancing changes between two political ideologies: when one ideology becomes more radical or extreme the other ideology becomes less so. Balancing should be contrasted with changes where both ideologies become more or become less

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17 Eigenvalues provide concise and useful information about transformations of political space, and extensive use of them is made below. Eigenvalues are real or complex numbers associated with square matrices. A square matrix $A$ of dimension $n$ has exactly $n$ eigenvalues. These numbers are the solutions to the polynomial equation $det [A - \lambda I] = 0$. The eigenvalues of skew-symmetric matrices are pure imaginary numbers, while the eigenvalues of orthogonal matrices all have an absolute value of one.
extreme. The direction of such a compensating change, to use a spatial analog, is orthogonal to the
vector defined by current political position, which is why it generates political rotation.

Related to but not quite the same as balancing is identity maintenance. This process reflects the
reality defining role of the political center and the existence of political identities defined by distance
from it. A deviation defined identity of this kind is maintained when the political position changes, but
distance from the political center remains constant; that is by rotation. An example of this might be the
well known path from radical left to radical right (or vice versa) while avoiding the political center.
This political transformation involves reversing political ideologies, and is constructable by rotation
whenever the number of dimensions to be reversed is even.

Now consider the three canonical distance altering transformations of political space: unification,
dispersion, and reconfiguration. As usual, we will focus upon the simplest distance altering forms
allowing complexity to arise through combinations of various canonical transformations. Hence we
assume that the infinitesimal generator $A$ is both non-singular and symmetric. Non-singularity implies
that all the political dimensions under consideration are important and that none can be discarded
without distorting the analysis.\(^{18}\) It also means that none of the eigenvalues of $A$ is zero. Symmetry
implies that the transformation under consideration involves no rotation. Rotation can be introduced
later by combining an orthogonal component with a symmetric one. Symmetry also means that all the
eigenvalues of infinitesimal generator $A$ are real numbers (none is complex). The three distance altering
transformations will be differentiated by the eigenvalues of $A$.

\(^{18}\) Non-singularity implies that $A^{-1}$ exists, but it does not imply the absence of an autonomous political
subspace. The existence of an autonomous subspace means that some of the political dimensions can be analyzed
without considering the others. Non-singularity, on the other hand, means that all dimensions must be included if all
of the dimensions are to be analyzed.
In general terms, a political unification occurs when all positions in the political space move closer to the political center. Such a transformation might happen during the opening phase of a politically popular foreign war (e.g. World War I in Germany or France) or after an attack upon the country (e.g. Pearl Harbor or 9/11). A canonical political unification is a constructable transformation whose infinitesimal generator is symmetric and has only negative eigenvalues.\(^1\) It follows that the matrix exponential of a canonical political unification \(e^{tU}\) will also be symmetric. Furthermore, its eigenvalues will be real, positive, and less than one.\(^2\) Political unification is volume decreasing. The volume of any finite region of political space is reduced by the transformation, and the reduction factor is identical for all finite regions.

In its canonical form, the political unification process is equivalent to a set of linear contractions towards the origin. The change in each position of the political space can be decomposed into precisely these linear contractions. The number of directions in which the contractions occur equals the number of dimensions in the political space (because \(A\) is non-singular), and the directions of contraction are perpendicular to each other (because \(A\) is symmetric). However, the directions in which the linear contractions occur usually differ from the axes of the political space. These directions are given by the eigenvectors of \(A\) (which are also eigenvectors of \(e^{tU}\)), and the rates of contraction equal the absolute values of the eigenvalues associated with these eigenvectors. Thus the most pronounced contraction towards the political center occurs in the direction specified by the eigenvector corresponding to the most negative eigenvalue.

\(^{1}\) The existence of an infinitesimal generator is a sufficient condition for constructability. If a transformation has an infinitesimal generator it is redundant to declare it constructable.

\(^{2}\) All canonical political unifications of the same dimension are topologically equivalent (Arnold 1992, p. 199). Thus the trajectories that occur in any two such transformations can be mapped into each other without ever crossing. Canonical political polarizations of the same dimension are also topologically equivalent.
A political dispersion is the exact opposite of a political unification. It happens when all positions in the political space move away from the political center. Political dispersion could be generated by an economic, political, or cultural crisis (e.g. the Great Depression) and, in extreme cases, it can lead to the disintegration of society. But it will rarely induce a civil war because the separating positions move in different directions. A canonical political dispersion is defined by its infinitesimal generator $A$. The latter is symmetric but, in contrast to political unification, has only positive eigenvalues. Consequently all eigenvalues of a dispersing matrix exponential $e^{At}$ exceed one, and the transformation is volume expanding.\(^\text{21}\) The first example in section four (6) is a very simple two-dimensional political dispersion.

If the word “contraction” is replaced by “expansion”, then everything said above about decomposing political unifications applies to dispersions as well. Thus the canonical political dispersion is an expansion in the $n$ directions defined by the eigenvectors of $e^{At}$. The amount of expansion in each direction is specified by the associated eigenvalue, with the largest expansion being in the direction associated with the largest eigenvalue. Because the expansions in different directions are typically of different magnitude, a region in political space changes shape as well as increases volume under a dispersing transformation.

The third and final distance altering transformation is political reconfiguration also called asymmetric dilation. The essential difference between reconfiguration and unification or dispersion is that the infinitesimal generator of the former has both positive and negative eigenvalues. This means that expansions occur in some directions (those corresponding to positive eigenvalues), while contractions occur in other directions (those associated with negative eigenvalues). The canonical

\(^{21}\) This follows from Liouville’s formula (11).
political reconfiguration, like the other two distance altering transformations, can be decomposed into these linear dilations. Because it is expanded in some directions and contracted in others, virtually any region in political space will be drastically re-shaped by such a transformation. In the real world, political reconfigurations occur more frequently than either contractions or expansion. The third example in section four (8) is a two dimensional reconfiguration. The eigenvalues of its infinitesimal generator are +1 and -1.

A political reconfiguration can be volume expanding, contracting, or (rarely) preserving depending on whether the trace of infinitesimal generator $A$ is positive, negative, or zero.\textsuperscript{22} When the contractions and expansions are sufficiently pronounced, a reconfiguration establishes a de facto redefinition and simplification of the political space. The dimensions of political space are implicitly redefined to coincide with the directions of expansion and contraction (which remain orthogonal to each other). Dimensions along which pronounced contraction occurs lose political relevance because political positions do not differ much when measured in these directions. On the other hand, political positions become sharply differentiated or even polarized along the vectors of expansion. By reducing the number of salient dimensions, such reconfiguration constitutes an implicit simplification of political space.\textsuperscript{23} As we shall see, something of this nature happens to American political space between 1982 and 2002. The emergence of extreme political conflict is likely to be associated with a reconfiguring

\textsuperscript{22} In contrast to canonical unifications and dispersions, not all canonical political reconfigurations of the same dimension are topologically equivalent. Canonical reconfigurations of the same dimension are topologically equivalent if and only if their infinitesimal generators have the same number of positive eigenvalues (Arnold 1992, p. 199). Assuming symmetry and non-singularity, these matrices will also have the same number of negative eigenvalues.

\textsuperscript{23} This simplification process bears a formal similarity to principal components analysis, but the matrix exponential, central to the analysis of political space transformation, is not relevant to principal components analysis (Gregg and Banks 1965, Dunteman 1989).
transformation of political space (McAdam, Tarrow, and Tilly 2001).

As indicated at the start of this section, every constructable political transformation is a combination of translation, rotation, and one of the three distance altering dilations (unification, dispersion, reconfiguration). And, conversely, every combination of these canonical transformations is itself constructable. The conceptual discussion of political space and its transformation is now complete. We now apply these concepts to a real historical political transformation. Empirical analysis of political space transformations involves determining (a) what kind of transformation has occurred, (b) separating the transformation into constructable and non-constructable parts (c) decomposing the constructable parts into canonical processes, and (d) identifying the discontinuities that make some parts non-constructable. The next section outlines some methods for making these empirical analyses.

7. Estimating Transformations

Estimation methods may be conveniently divided into three sequential parts: dimension measurement, interval estimation, and generator determination. Dimension measurement involves scaling of the political dimensions that have been deemed relevant to the political system. It also means gathering information about all of these dimensions for a sufficient number of suitable cases. Information about each case must be gathered for both the start and the end of the relevant time interval. The number of cases used must equal or exceed the number of political dimensions.

The political space transformation model (1) and (2) implies that an affine transformation (13) sends political vectors at the start of the time interval $X(0)$ into political vectors at the end of the time interval $X(t)$.
Interval estimation entails approximation of both the interval transformation matrix $C_i$ and the interval translation vector $D_i$. Generator determination involves moving from the interval time frame to the instantaneous time frame. More specifically, it involves deciding whether or not the results of interval estimation in (13) are constructable. If the affine transformation above is constructable, we then estimate the infinitesimal generator $A$ that yields $C_i$ and the infinitesimal vector $B$ that yields $D_i$. Using (9)

(14) \[ C_i = e^{At} \quad \text{or} \quad A = \frac{1}{t} \log[C_i], \]

where $\log[C_i]$ indicates the matrix logarithm of $C_i$. Expression (9) also implies that

(15) \[ D_i = A^{-1}[e^{At} - I]B \quad \text{or} \quad B = [e^{At} - I]^{-1} A D_i. \]

The following discussion avoids the really thorny estimation problems by considering only the generic situation, that is the situation that almost always happens. This is not a practical limitation on the estimation procedures presented below. If, by some anomaly, the generic situation does not exist, it can be produced by adding a few more cases. An example of this approach is expression (15) above which assumes the infinitesimal generator $A$ to be non-singular.

Consider the problem of dimension measurement. Suppose that the number, identity, and conceptual meaning of the space defining political dimensions are known. Suppose also that the

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24 If the transformation is not constructable, we change it in a simple way to make it constructable. Using this constructable variation, the process is projected forward in time. At some designated time of discontinuity these changes are reversed, and the outputs of the transformation process are modified accordingly. Finally the revised output is projected forward in the same way as before. See section nine.

25 For a sustained effort to deal with non-generic problems in the realm of generator determination see Singer and Spilerman (1976).
dimensions salient at the start of the interval as the same as those salient at the end of the interval.\textsuperscript{26} Any institution or collectivity whose location on each of the salient political dimensions can be measured both at the start and at the end of the relevant time interval is called a political case. This might be a political party, corporation, labor union, or newspaper. A political case might also be a social class, religious denomination, occupational group, or urban neighborhood. The important point is the feasibility of measuring the location of a political case within the political space. As mentioned above, the number of political cases must equal or exceed the number of political dimensions. Linear independence is critical for estimation. Both at the start and at the end of the time period considered, the number of linearly independent political cases must equal the number of political dimensions.\textsuperscript{27}

Political location on each dimension is measured on a scale ranging from $-s$ to $s$, where $s$ can be any integer but should be identical for every political dimension.\textsuperscript{28} The zero point of the scale corresponds roughly to the conceptual midpoint of the political dimension and, wherever possible, left of center locations are coded with negative numbers and right of center locations with positive numbers. Distribution on any dimension is not standardized, but reflects the prevailing shape of political space. Thus the amount of variation will typically differ among the dimensions with extreme positions more common on some than on others. Scaling can be accomplished by various methods, but must be applied consistently. Cases can be obtained from various sources – survey data, historical records, policy statements, political practices – but each dimension must be coded on the same scale. Dimension measurement is the least systematic of the estimation methods triad mentioned above, with procedural

\textsuperscript{26} The situation where old dimensions loose salience or new dimensions gain salience can be analyzed by the conceptual apparatus proposed in this paper, but different estimation techniques are required.

\textsuperscript{27} Since the dimension of the political position vector equals the number dimensions, the number of linearly independent cases obviously cannot exceed the number of dimensions.

\textsuperscript{28} Although for purposes of standardization political location is scaled to be within the $[-s, s]$ interval at the times of measurement, it need not and often does not remain within this interval.
advice amounting to little more than general guidelines. Nevertheless the value of the entire analysis depends heavily upon the effectiveness of dimension measurement. Bear in mind, however, that political space is a theoretical construct and not what Bourdieu refers to as a “subjective representation” of common sense knowledge (1989, p.15). Nor is the population of political cases rigorously defined. The latter consists of any social entity with a measurable political position that should be transmuted in the manner specified by the putative transformation process. The entire estimation process is more robust if the political cases used have widely scattered political positions.

Now consider interval estimation, the second leg of the estimation triad. We use the least squares principle to estimate the transformation matrix \( C \) and the translation vector \( D \) from the data provided by dimension measurement. From expressions (14) and (15) it is evident that \( C \) and \( D \) are both functions of time interval length. For an interval of given length, they are chosen to minimize the squared error that results when a linear rule (i.e. an affine transformation) generates political position at the end of the interval from political position at the beginning.

With \( d \) political dimensions and \( n \) political cases dimension measurement procedure produces two \( d \times n \) data matrices, one for the start of the interval (time 0) and one for its end (time \( t \)). These matrices have the following form:

\[
X_s = \begin{bmatrix}
    x_{11}(s) & x_{12}(s) & \cdots & x_{1n}(s) \\
    x_{21}(s) & x_{22}(s) & \cdots & x_{2n}(s) \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{d1}(s) & x_{d2}(s) & \cdots & x_{dn}(s)
\end{bmatrix}
\]

(16) \quad s = 0, t

where \( x_{ij}(s) \) is the location of case \( j \) on dimension \( i \) at time \( s \). The rows of matrix \( X_s \) correspond to dimensions (\( i \) through \( d \)), while the columns correspond to political cases (\( j \) through \( n \)). Define the
$d+1 \times n$ dimension matrix $K$ as

\begin{equation}
K = \begin{bmatrix}
X_0 \\
1_n
\end{bmatrix}
\end{equation}

where $1_n$ is an n-dimensional row vector of ones. The least square principle (Draper and Smith 1966, pp. 58-9) yields the following estimate of transformation matrix $C$, and translation vector $D$,

\begin{equation}
[C_t, D_t] = X_tK^T(KK^T)^{-1}.
\end{equation}

If the number of cases exactly equals the number of dimensions and the cases are linearly independent, then these estimates of $C$, and $D$, exactly reproduce $X_t$ through the affine transformation in (13).

The third phase of estimation, generator determination, involves rather difficult technique, but our genericity assumption simplifies things greatly. The first step in generator determination is deciding whether the transformation matrix $C$ is constructable; that is deciding whether an infinitesimal generator $A$ with only real elements exists such that the equations in (14) are satisfied. The generic approach allows us to assume that all the eigenvalues of matrix $C$ are different. In the unlikely event that the eigenvalues are not unique, then addition of a few more political cases will correct the situation. If all the eigenvalues do differ – a condition we henceforth assume – then a necessary and sufficient condition for the constructability of transformation matrix $C$ is non-singularity and absence of negative real eigenvalues (Culver 1966).

Even when transformation matrix $C$ is constructable, the infinitesimal generator $A$ that produces it is usually not unique. Excluding multiple eigenvalues, it can be shown that uniqueness holds if only if all the eigenvalues of $C$ are real and positive (i.e. none are complex). In other words, the generator is
unique only when the transformation involves dilation but no rotation.\textsuperscript{29} But without multiple eigenvalues the absence of uniqueness is not a major estimation problem. Complex eigenvalues create difficulties because their logarithms are not unique. When taking the logarithm of complex eigenvalues, we simply choose the principal value thereby minimizing the amount of rotation involved in the transformation.\textsuperscript{30}

To estimate infinitesimal generator $A$ we obtain $\text{Log } C$, and divide this matrix by the length of the time interval $t$ (see (14)). The first step in calculating $\text{Log } C$ is converting matrix $C_{i}$ into Jordan block form with an appropriate similarity operator $S$

\begin{equation}
C_{t} = SJS^{-1}
\end{equation}

where $J$ is a quasi-diagonal real matrix of the following type

\begin{equation}
J = \text{Diagonal}\{\lambda_{1}, \ldots, \lambda_{j}, \Lambda_{1}, \ldots, \Lambda_{k}\}.
\end{equation}

The $\lambda_{i}$ above correspond to the real valued eigenvalues of matrix $C_{i}$ which, by the constructability of $C$ and the genericity assumption, are all positive and different. The $\Lambda_{i}$ are $2 \times 2$ real matrices that correspond to conjugate pairs of complex valued eigenvalues $u+iv$ and $u-iv$. These matrices have the form

\begin{equation}
\Lambda = \begin{bmatrix}
u & v \\
-v & u
\end{bmatrix}
\end{equation}

and they are all different. Since $J$ is quasi diagonal all its other elements are zero. It is always possible

\textsuperscript{29} Translation is irrelevant to the existence or uniqueness of constructability.

\textsuperscript{30} It is often convenient to write complex numbers in polar coordinates: $z = x + iy = re^{i\phi}$, where $r = \sqrt{x^{2} + y^{2}}$ and $\phi$ is an angle such that both $x/r = \cos \phi$ and $y/r = \sin \phi$. The non-negative real number $r$ is called the radius of the complex number $z$, and the angle $\phi$ is called the argument of $z$. If $-\pi < \phi < \pi$ then $re^{i\phi}$ is said to be the principal value of $z$. The exponential of $z$ is $e^{z} = e^{(\cos y + i \sin y)}$. The natural logarithm of $z$ is $\ln z = \ln r + i\phi$. The principal value of $\ln z$ is written $\text{Ln } z$. 

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to convert a real matrix into Jordan block form, and the procedures for doing so are more or less equivalent to finding its eigenvalues and eigenvectors.

It is easy to show that

\[ \log C = S (\log J) S^{-1} \]

and

\[ \log J = \text{Diagonal} \{ \log \lambda_1, \ldots, \log \lambda_j, \log \Lambda_1, \ldots, \log \Lambda_k \} \]

If \( u \pm iv = r e^{\pm \phi} \) then the logarithm of the corresponding \( 2 \times 2 \) matrix \( \Lambda \) is

\[ \log \Lambda = \log \begin{bmatrix} u & v \\ -v & u \end{bmatrix} = \log \begin{bmatrix} r \cos \phi & r \sin \phi \\ r \sin \phi & r \cos \phi \end{bmatrix} = \begin{bmatrix} \log r & \phi \\ -\phi & \log r \end{bmatrix} \]

where \(-\pi < \phi \leq \pi\). This enables us to compute the principal value of the infinitesimal generator \( A \).

One of the genericity assumptions is the non-singularity of matrix \( A \). This is equivalent to assuming that no eigenvalue of \( C \) equals one. If this is so, it follows that matrix \( [e^u - I]^{-1} \) exists, and (15) can be used to estimate the infinitesimal vector \( B \). This completes the estimation process. Empirical investigation of real political space transformations can now proceed.


The transformation of United States political space over the two decades between 1982 and 2002 was analyzed using data from the General Social Surveys for 1982, 1993, and 2002. This twenty year interval was divided into two sub-intervals, 1982 to 1993 and 1993 to 2002, and each of these sub-intervals was analyzed separately. Important transformations of political space can occur within a ten year interval, but the time lag is short enough to justify the basic model proposed above. By comparing political transformations over two consecutive intervals, we gain perspective on the nature of such
transformations and, as it turns out, can examine the difference between constructable and non-constructable transformations. The General Social Survey provides information about many, though certainly not all, political dimensions. The consistency of its questions greatly facilitates comparisons between 1982, 1993, and 2002.

All questions pertaining to politics, included on all three of the General Social Surveys in question, and answered by a sufficiently large number of respondents were examined. These three requirements greatly reduced the number of relevant questions. It proved possible to construct meaningful and temporally consistent scales on seven important political dimensions pertaining respectively to class, race, gender, civil liberties, crime, military affairs, and the environment. These seven dimensions do not cover the entire realm of American politics. They are, however, sufficiently salient, differentiated, and numerous to provide an intelligible if condensed representation of American political space. Indeed, we shall assume that these seven dimensions constitute an autonomous subspace.\(^3\) Each dimensions can be arrayed along a left-right continuum with the negative direction signifying “leftness” and the positive direction signifying “rightness”. The meaning of these seven political dimensions is further delineated in Table 1. [[ Insert Table 1 about here]].

The next step was to identify coherent social aggregates whose collective location on each of the seven political dimensions in 1982, 1993, and 2002 could be measured. The “collective location” requirement was operationalized to mean that, in each of the three years, not fewer than 20 members of the group (and usually a lot more) answered all the questions included within a particular political scale. On each dimension the group was assigned a political location defined by the mean of its individual members. These means were standardized using 1982 sample means and standard deviations so that

\(^3\) See expression (12) and the surrounding discussion.
overall translations of political space could be measured. By these procedures a seven dimensional political position was measured for each social aggregate in 1982, 1993, and 2002. After carefully examining all the variables available on the three General Social Surveys, thirty nine suitable social aggregates were identified. These groups are listed in Table 2. Information about their political positions in 1982, 1993, and 2002 constituted the basic data for analyzing the transformation of American political space. [[[Insert Table 2 about here]]].

The third step involved estimating political space transition matrices and transition vectors from 1982 to 1993 ($C_1$ and $D_1$, respectively), from 1993 to 2002 ($C_2$ and $D_2$), and from 1982 to 2002 ($C_3$ and $D_3$). In each case, this is done by regressing the political dimension variables for the later year upon those for the earlier year using vector adaptations of ordinary least squares methods, that is using expressions (17) and (18) above. The results are presented in Table 3. [[[Insert Table 3 about here]]]. The transition vectors $D_1$, $D_2$, and $D_3$ are easy to interpret. They indicate translations of the entire political space. That is, over the time interval indicated, each position in American political space is incremented in the manner specified by the relevant transition vector. Of course translation is not the only change visited upon a political position (otherwise $C_1$, $C_2$, and $C_3$ would be identity matrices and hence superfluous), but it is a vital component of the overall transition process. Between 1982 and 1993 transition vector $D_1$ shows that American political space was translated in a generally liberal or leftward direction. The largest liberalizing translations occurred in the gender and civil liberties dimensions. The one significant exception was class ideology which was translated in a rightward or conservatizing direction. That labor union membership declined from 21% to under 16% of the American workforce may be germane to this

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32 Statisticians sometimes distinguish between a model and a population (Flury 1997, p. 209). The concept of a model is appropriate in the present context. We are trying to estimate the properties of a political transformation process, but make no claims about the specific population to which the model applies.
conservatizing translation.

The translation of American political space between 1993 and 2002 is even more striking. Transition vector $D_2$ shows that small leftward translations occurred on seven of the eight political dimensions. However, the dramatic translation is the single exception to this general liberalizing movement. Military affairs ideology took a great leap towards the right; by far the largest political translation detected anywhere in the present analysis. This conservatizing leap probably reflects the impact of 9/11 on American political space. As we shall see, the impact of 9/11 is also relevant to the non-constructability of the political space transformation between 1993 and 2002.

The transformation matrices $C_1$, $C_2$, and $C_3$ are more difficult to interpret. A good place to start is by examining the elements on the main diagonals. These elements indicate the transforming impact of a political dimension upon itself. We ordinarily expect these impacts to be positive and relatively large. A normal self transforming impact is defined as one that is positive and larger in absolute value than the impact of any other dimension. In other words, the diagonal element corresponding to the dimension is positive ($c_{ii} > 0$) and larger in absolute value than any other element in that row of the transformation matrix ($|c_{ii}| > |c_{ij}|, i \neq j$). From 1982 to 1993 the self impact of all dimensions was positive and six of the seven political dimensions were normal (see matrix $C_1$). The situation between 1993 and 2002 is quite different. Now two dimensions have a negative self impact, and only two of the seven – civil liberties ideology and environment ideology – are normal. This suggests considerable disparity between the two political space transformations being analyzed.\(^{33}\)

\(^{33}\) Transition matrix $C_j$ and transition vector $D_j$ help us evaluate the validity of the model and methods used in this analysis of political space transformation. If all assumptions were fully satisfied, then the following equalities would hold: $C_j \times C_j = C_j$ and $C_j \times D_j + D_j = D_j$. These equalities, as the reader may check, are closely but not exactly satisfied.
Next we examine the volume changing properties of the two political space transformations. This is done by calculating the determinants of the transformation matrices. Since all these determinants are less than one in absolute value, the transformations considered here are volume contracting. As explained above, this means that every region of political space becomes a region of smaller volume, the ratio of volume reduction being measured by the determinant of the transformation matrix. But it does not imply that all political positions draw closer together. Between 1982 and 1993, political space in the United States contracted but also elongated. In effect the space simplified—the number of practically meaningful political dimensions decreased—and stretched out over fewer and different dimensions. The dimensions defined by the transformation were composites of the original political dimensions. Thus distances between pairs of political positions typically increased, but the positions themselves were distributed around a smaller number of composite dimensions.

Deeper insights into these transformations of political space can be obtained from the eigenvalues of the transformation matrices $C_1$, $C_2$, and $C_3$, which are given in Table 4. Inspection shows that for each matrix no two eigenvalues are equal, which is the generic situation for empirically generated matrices. More importantly, matrices $C_2$ and $C_3$ each have two negative eigenvalues, implying that the transformations of political space from 1993 to 2002 and from 1982 to 2002 are not constructable. On the other hand, transformation matrix $C_1$ has no negative eigenvalues indicating that the political space

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34 The measure of volume depends upon the number of dimensions. In this case volume means volume as measured in a 7-dimensional Euclidean space.

35 The rapid contraction of political space implied by these determinants is troublesome. If systematic transformation of political space really entails such precipitous contraction, it could not reproduce political space in anything like its current form. As suggested in the concluding section, this could mean that contingent processes have a critical role in reproducing political space. It is also possible that the seven dimensions being studied do not constitute an autonomous subspace, or that the dimensions of American political space are themselves changing. In the latter case, discontinuation of antiquated political dimensions would yield ostensible space contraction.
transformation from 1982 to 2003 is constructable, and hence the changes involved could have happened in arbitrarily small and homogeneous increments. The remainder of this section discusses the constructable transformation from 1982 to 1993. The non-constructable transformation from 1993 to 2002 is analyzed in section nine.

Examination of the eigenvalues of $C_t$ reveals the structural composition of the 1982 to 1993 space transformation. In addition to the translation discussed above, the transformation consists of two linear expansions (i.e. stretching along a straight line), the largest by a factor of 1.5; three linear contractions (i.e. shrinking along a straight line), the smallest by a factor of .03; plus a centripetal planar rotation (i.e. two dimensional spiraling inward) through an angle of $37^\circ$. Of these six superimposed processes, the two expansions have by far the greatest impact. They suggest that, over the eleven year interval considered, political space moved towards becoming a two dimensional structure, and that the implied reduction in the volume of occupied political space (equal to the product of all eigenvalues) was precipitous. And if the political space transformation that happened between 1982 and 1993 continued over a long period of time (say a century) American political space would become effectively one dimensional, that being the dimension corresponding to the largest positive eigenvalue. We call this dimension the *shadow destiny* of the transformation because it defines the implicit direction in which things are heading. Further discussion of the 1982 to 1993 shadow destiny is postponed until after considering the generative core of this political space transformation.

The infinitesimal generator $A_t$ and the infinitesimal vector $B_t$ for the 1982 to 1993 political space transformation are given in Table 5. The elements of the infinitesimal generator specify exactly how current political position influences the rates at which that political position changes. Current political position, in the situation being analyzed, has seven
dimensions or components. The seven numbers in row \( I \) of infinitesimal generator \( A \), specify the linear impacts of these seven components on the rate at which component \( I \) changes. Similarly, the seven numbers in column \( j \) of \( A \), specify the linear impacts of component \( j \) on the change rates of the seven political position components. Despite their unfamiliarity, the infinitesimal generator and infinitesimal vector provide a sharp image of the underlying transformation process. Whereas the transformation matrix and transformation vector represent the aggregate of many different effects, the infinitesimal matrix and infinitesimal vector depict direct, instantaneous, and uncombined influences. Through the process depicted in expression (9), the infinitesimal structure \( A \) and \( B \) reproduce the transition structures \( C \) and \( D \) exactly.

Five of the seven elements on the main diagonal of \( A \) are negative, indicating that the direct effect of political position components on themselves tends to be contracting, that is towards the political center. The self contracting influence of environmental ideology on itself is considerable and far exceeds the self contracting influence of any other component. The 1982 to 1993 transformation has a single equilibrium (given in Table 6) with clearly left of center positions on gender and civil liberties, mildly left of center positions on military affairs and environment, and mildly right of center positions on class, race, and crime. However, this equilibrium position is unstable and substantively irrelevant: almost all positions near the equilibrium move away from rather than towards it.

A better intuitive grasp of the how the transformation works is gained by considering its influence on the central or origin position of the political space in 1982. The inferred or projected trajectories of

\[ \text{The components of a political position will also be called locations, elements, or variables when the context makes the meaning clear. The term dimension usually refers to the entire range of possibilities rather than the specific location within this range.} \]

\[ \text{The trace of } A \text{ is negative (-.498) because the 1982 to 1993 transformation is space contracting.} \]
all seven components of the central position are depicted in Figure 3. The top graph shows the projected evolution of the central position over the eleven year interval between 1982 and 1993. The bottom graph in Figure 3 projects this same process over a 100 year interval showing the long term directions in which the transformation, as modeled, would take the central position of 1982. Two features of the top graph stand out. The amount of change in the components of the central position over the eleven year interval is always small. None of the seven components changes by more than 0.5 units. Secondly, the seven components are fairly evenly dispersed over the interval [-0.5,+.05]. No obvious clustering of components has occurred.

The projected evolution of the 1982 central position over a full century shows quite different characteristics: considerable clustering happens, and some components depart greatly from their initial position. Three distinct clusters are apparent. Ideologies pertaining to race, civil liberties, and military affairs move slightly in a conservative or right direction. Environmental ideology goes moderately in a liberal or left direction. Ideologies about class, gender, and crime move sharply towards the left. This particular pattern defines one pole of the shadow destiny for the 1982 to 1993 political space transformation.

Another way of understanding this seven dimensional transformation is by examining how it effects relations between four prototypical political positions: consistent left, consistent right, center, and mixed left and right.\textsuperscript{38} Two ways of describing the relations between political position are (a) the distance between the positions and (b) the angle between vectors representing the positions.\textsuperscript{39} Although these

\textsuperscript{38} These political positions were represented by the following vectors: consistently left (-3,-3,-3,-3,-3,-3); consistently right (3,3,3,3,3,3); center (0,0,0,0,0,0,0); mixed (-3,-2,-1,0,1,2,3).

\textsuperscript{39} If the position vectors point in the same direction (starting from the origin) the angle between them is zero radians. If the vectors are orthogonal to each other the angle between them is $\pi/2$ (about 1.57) radians. If the position vectors point in opposite directions the angle between them is $\pi$ (about 3.14) radians. The greater the angle
relational properties are not independent, each provides distinctly different information. The four prototypical political positions yield six different pairs. Figure 4 diagrams how relations between these pairs evolve over the 1982 to 1993 political space transformation. The top graph in Figure 4 shows the distances between the political position over the eleven year interval. The bottom graph represents the angles between the two position vectors of each pair. [[Insert Figure 4 about here]].

Figure 4 indicates that the consistently left position is implicated in all major relational changes. Consider relations between the consistently left and the consistently right positions. Both the distance and the angle between these two positions remain large over the entire 1982 to 1993 interval. Consider relations between the consistently left and the mixed positions. In this case both the distance and the angle between the positions start at moderate levels but increase rapidly over the eleven year period. With regard to the left and center political positions, distance increases but angle remains relatively constant. On the other hand, all position pairs not involving the left remain relatively stable on both distance and angle between over the period considered. A partial exception to this claim is the angle between the center and mixed positions which drops rapidly over the first two years after 1982, but then increases gradually and by the end of the interval is only 30° less than its starting position. Taken as a whole, the 1982 to 1993 political space transformation tends to differentiate consistently left from most other political positions.

This phenomenon is clarified by examining the asymptotic or long term behavior of the transformation model, what we call its shadow destiny. The defining characteristic of the shadow destiny between vectors, the greater the difference between the political positions these vectors represent.

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40 The trajectory of the distance between the left and center positions is exactly the same as that between the left and mixed positions. Thus the graph of the latter covers the graph of the former, which is why only five trajectories are shown on the top part of Figure 4. Note that the angle between left and center positions follows a different trajectory than the angle between the left and mixed positions.
is an extreme simplification of the political space. No matter how many dimensions are initially present, the space becomes increasingly one dimensional over time. Within this one dimension all components either diverge towards infinity or converge towards the single equilibrium of the generic political space. When divergence occurs, as in the present situation, the components of political space become clustered into different groups each one of which approaches the same political extreme at a somewhat similar rate. The components of a group can approach either a right (positive) or a left (negative) political depending upon the starting position in the political space, but the components of the group always move together. We will describe the shadow destiny mainly by identifying the component clusters that it generates.

The shadow destiny of the political space seldom or never comes about. The dynamics of political space change too much for this to happen. As the name suggests, it constitutes an implicit rather than a realized political future. The importance of the shadow destiny derives from its potential impact upon political awareness. Politically alert actors often sense the direction in which political space is moving long before the destination is approached. Intuitions or apprehensions about the future can influence how political actors behave in the here and now thus impacting the dynamics of political space.

The general nature of the shadow destiny can be assessed from the eigenvalues of the infinitesimal generator $A$, which are given in Table 6. Two of the seven eigenvalues have positive real parts indicating that the shadow destiny involves divergence rather than convergence. These eigenvalues also imply that the transformation would eventually compress the seven dimensional political space into a two dimensional subspace. The eigenvectors associated with these two positive eigenvalues (also given in Table 6) determine this two dimensional subspace. The eigenvector corresponding to the largest positive eigenvalue – named the dominant eigenvector – defines the one dimensional shadow destiny of the system. The eigenvector associated with the other positive eigenvalue is called sub-
An eigenvector remains an eigenvector when it is multiplied by a non-zero number. This means that eigenvectors are unique only up to scalar multipliers. The relative size of its elements is a structural property of an eigenvector, but the sign of the elements can be reversed through multiplying by -1.

In this particular case, the dominant and sub-dominant eigenvectors are quite similar. In fact the angle between them is less than 16°, meaning that compression towards the one dimensional shadow destiny occurs relatively quickly. The dominant eigenvector has three large positive components (corresponding to class, gender, and crime), one positive element of moderate size (corresponding to environment), and three relatively small negative elements (corresponding to race, civil liberties, and military affairs). These relative magnitudes define the shadow destiny clusterings. Because the 1982 to 1993 political space transformation produces divergence rather than convergence, two different shadow destinations – mirror images of each other – exist. In one of these polar destinations (shown in the top graph of Figure 5) class, gender, and crime ideologies move sharply to the right; environment ideology moves moderately to the right; while race, civil liberties, and military ideologies move slightly to the left. Because the greatest movement is towards the right, we call this the right pole. The opposite or inverse polar destination has already been depicted on the bottom half of Figure 3. Here the main thrust is towards the left, so we call this the left pole.

What is the relationship between initial position in the political space and polar destination? Because the model of space transformation is deterministic, initial position fully determines polar destination. The basic pattern can be seen in the bottom graph of Figure 5. This plots the angles between the right pole and the vectors defined by the four prototypical political trajectories (consistent left, consistent right, center and mixed) over 100 years. If this angle approaches zero, then the trajectory is moving in the direction of the right pole. If the angle approaches π, then the trajectory moves in the

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41 An eigenvector remains an eigenvector when it is multiplied by a non-zero number. This means that eigenvectors are unique only up to scalar multipliers. The relative size of its elements is a structural property of an eigenvector, but the sign of the elements can be reversed through multiplying by -1.

42
direction of the left pole. [[Insert Figure 5 about here]]

Not surprisingly, the consistent left position transforms towards the left pole, while the consistent right position transforms towards the right pole. Figure 5 also shows that the center position moves towards the left pole, while the mixed position moves towards the right pole. These four cases indicate the general pattern. Initial positions decisively and uniformly on the left are transformed towards the left pole, while initial positions decisively and uniformly on the right eventually move towards the right pole. The destinations of intermediate positions are difficult to predict, and I have found no general rule indicating what they will be.


As indicated above, \( C_1 \) the transformation matrix for the nine year period between 1993 and 2002 has two negative eigenvalues (see Table 4) and is thus non-constructable. This means that the estimated transformation of political space could not happen through a continuous evolution and must involve some kind of discontinuity or, more exactly, reversal of the political space. Although the methods used in the previous section require modification, analysis of the 1993 to 2002 transformation is still possible. Before proceeding further, we must gauge the nature of the political discontinuity that occurs between 1993 and 2002. Using this information we can form a constructable variation of the observed transformation.

An elementary reversal is a change that (a) affects the transformation of political space in a straightforward manner, (b) is readily invertible, and (c) changes the transformation matrix from constructable to non-constructable. Elementary reversals are of two basic kinds: input or column reversals, and output or row reversals. An input reversal reverses the signs of one or more columns of
the transformation matrix or, equivalently, reverses the signs of the input variables corresponding to these columns. An input reversal essentially flips the impact of the designated dimensions on how all dimensions are transformed. An output reversal reverses the signs of certain rows of the transformation matrix or, what amounts to the same thing, reverses the signs of the output variables associated with these rows. An output reversal flips the results of transformation on the specified dimensions.

The simplest elementary reversal that could change $C_2$ into a constructable matrix involves reversing (i.e. multiplying by minus one) either two rows or two columns of this transformation matrix. With seven dimensions in the political space, there are exactly 42 such elementary reversals. It turns out that only five of these reversals (all of them input reversals) change $C_2$ into a constructable matrix having no negative eigenvalues. The largest eigenvalue of $C_2$ exceeds one, indicating that the transformation process diverges from equilibrium rather than converging towards it. Requiring that the elementary reversal be divergent eliminates all possibilities but one: that which reverses columns one and five of transformation matrix $C_2$. This elementary reversal flips the impact of the class and crime dimensions and is the one used in the analysis below. Of the five candidates, the class-crime input reversal entails by far the least amount of change from the observed transformation. We shall refer to it as the constructable variation of the observed transformation. The matrix for this constructable variation (labeled $VC_2$) as well as the eigenvalues of this matrix are given in Table 7 [[Insert Table 7 about here]]. Comparing the eigenvalues of $C_2$ with those of $VC_2$ shows that the main difference is a change from two negative eigenvalues into a pair of complex conjugates.

Exactly how does the non-constructable transformation of political space between 1993 and 2002 operate? We propose the following interpretation. The transformation is initially governed by the infinitesimal generator and infinitesimal vector corresponding to the constructable variation defined
above. At a certain point during the nine year interval a discontinuity or flip takes place. This involves suddenly reversing (multiplying by minus one) the input values of class and crime while keeping everything else constant. In effect this switch undoes the elementary reversal while maintaining the constructability of the process. After the discontinuity, the transformation process continues following the same infinitesimal generator as before. At the end of the time period this process yields the non-constructable transformation matrix originally observed. A graph of selected position trajectories shows sudden increments or saltuses (one for each dimension) at the time of discontinuity (see Figure 6).

The non-constructability of $C$ implies that a discontinuity in the transformation of political space occurs at some time point between 1993 and 2002, but gives no indication of exactly when this will be. Relying upon informal evidence and common sense, we shall locate the time of discontinuity at September 11, 2001. The destruction of the World Trade Center and the attack upon the Pentagon on that day is surely the most consequential political intervention during this nine year period. We analyze the political space changes induced by the discontinuity assuming them to be consequences of 9/11. This procedure rests upon strong suppositions and invites serious objections, but it does provide a quantitative estimate of how 9/11 impacted American political space. The intent to illustrate the power of the space transformation approach no doubt spurs methodological audacity.

The infinitesimal generator ($AC_2$) and the infinitesimal vector ($VB_2$) derived from the constructable variation ($VC_2$) along with eigenvalues of $AC_2$ are given in Table 7. They indicate that the constructable variation is volume contracting, and that it makes the political space increasingly one dimensional. The transformations of right and of mixed positions between 1993 and 2002 are diagramed in Figure 6. [[Insert Figure 6 about here]] The 9/11 discontinuity is evident in both diagrams (8 years after 1993). Being tantamount to a sudden reversal of the impacts emanating from the class and crime
variables, the nature of this discontinuity depends upon the initial values of these variables. The right and the mixed positions differ by six units on class ideology and by two units on crime ideology. Thus the impact of the 9/11 discontinuity differs for these two positions (e.g. the opposite directions of the gender ideology saltus in Figure 6).

Nevertheless the overall transformations of these two political positions between 1993 and 2002 show definite similarities. Over this interval, the crime and military affairs ideologies of both right and mixed positions become more conservative – a movement temporarily reversed by the 9/11 discontinuity – while the class and gender ideologies become decidedly more liberal. On the other hand race, civil liberties, and environmental ideologies retain center locations, or at least locations intermediate between the other two groups.

The long term impact of the 9/11 discontinuity is not evident in the diagrams of Figure 6 which end just one year after the event. In fact the short term impact can be directly opposite to the long term impact (e.g. crime and military affairs in Figure 6). Although assessing the long term consequences of the 9/11 must be a projection rather than a measurement, the assessment is certainly interesting and perhaps even useful. To appraise the long term impact of the 9/11 discontinuity we will compare the shadow destinies of the 1993 to 2002 political space transformation with and without this discontinuity.

The shadow destiny associated with the constructable variation is defined by the dominant eigenvector of $AC_2$ (given in Table 7) and consists of both a “right” and a “left” pole. The “right” pole, which has three distinct dimension or ideology groups: (a) class, race, crime, military affairs, and environmental ideologies move relatively rapidly to the right; (b) civil liberties ideology moves slowly to the right; (c) gender ideology moves slowly to the left. The “left” pole is just the reverse of the “right”
pole. Because the 9/11 discontinuity is equivalent to switching the values of input variables, it does not change the character of the shadow destiny. What it does change is the relation between initial position and destination pole. It also changes how rapidly an initial position moves towards a destination pole. For example, without the 9/11 discontinuity both the center left and the consistently left political positions eventually move towards the “left pole”. With the 9/11 discontinuity both these positions and every other plausible political position move fairly quickly towards the “right” destination pole.

The imputed evolution of the center political position can illustrate the generic effects of the 9/11 discontinuity. In Figure 7 the evolution of the of the center position is projected forward half a century after 1993 first with and then without the 9/11 discontinuity. Even without the 9/11 discontinuity, the center position would have moved towards the “right pole”. But the approach would have been gradual and only subsequent to a long phase of left-right balance among the seven constituent political ideologies. As the bottom graph in Figure 7 shows, the balance might have continued at least 42 years after 9/11. The discontinuity of 9/11 drastically alters the imputed trajectory of the center position in the four decades following destruction of the World Trade Center. Drastic polarization takes hold. Class, race, crime, military affairs, and environmental ideologies veer sharply to the right. Gender ideology moves slowly to the left. Civil liberties ideology, though still left of center after 42 years is about to turn right. Were this a real historical outcome, the entire political space would shift towards what was formerly considered right wing extremism.

That the events of 9/11 transformed political space towards the right will surprise almost no one. Considerable evidence suggests that the long term effects of terrorism usually proceed in this direction (Bonanate 1979, Hamilton and Hamilton 1983). That such a conservatizing transformation is implicit in data collected only one year after the events, data that on first interrogation reveals no such tendency,
is perhaps more remarkable. The capacity to reveal latent and barely expressed tendencies arises directly from the use of a dynamic model to analyze the transformation of political space. Yet this capacity, if mechanically applied, can lead to absurd conclusions. A combination of historical awareness and interpretive caution is needed to use the methods proposed herein convincingly.

10. Conclusion

The concept of political space as a structure or field distinct from political positions of the individuals who occupy that space is not new. It has often been used in a loose and metaphorical way by historians and political scientists studying processes of political change. For example in a recent study of the European left historian Geoff Eley examines how post-1968 social movements remade “national political space” (2002, p. 11). This paper moves beyond metaphorical usage by proposing a specific model of how political space changes over time. This dynamical model is used to formulate a typology of political space transformations and also to distinguish constructable from non-constructable transformations. Estimation methods are presented, and the model is then applied to political change in the United States over the last two decades of the 20th century.

As indicated at the outset, this paper focuses upon systematic rather than contingent transformations of political space. We establish that the systematic transformation of American political space between 1982 and 1993 is constructable and could have happened through gradual homogeneous evolutionary change. On the other hand, the systematic transformation that occurred over the next nine years is not constructable and entails a political discontinuity of some kind. The basic tendency in the earlier period featured a gradual simplification of political space into a smaller number of political dimensions plus a slight polarization in which consistently left positions pulled away from both
consistently right and mixed political positions. The discontinuity detected in the latter period induces a distinct rightward transformation of political space in which even consistently left political positions become more conservative. By associating the 1993-2002 discontinuity with the events of 9/11, we infer that the attacks on the Pentagon and the World Trade Center have had a pronounced conservatizing impact upon American political space.

Both empirical analysis and theoretical considerations suggest that systematic transformations of political space are likely to be political reconfigurations with perhaps an admixture of translation and rotation. If this is the case, then the principal thrust of systematic transformation is towards simplifying political space and reducing the number of political dimensions. Yet this contradicts both common sense and the everyday experience of political life. Both of these suggest that political space seldom undergoes a radical simplification of the sort latent within political reconfiguration. Something must counteract or at least mitigate the implied simplification. That something could be contingent (i.e. non-systematic) transformations of political space. It is certainly plausible that contingent events outside the framework of rule governed transformations should introduce new political dimensions and/or reinvigorate collapsing ones. The attacks of 9/11 certainly constitute such contingent events, but we have analyzed them within the framework of systematic transformations. Thus they are construed to impact the destination of political positions, but not the tendency towards simplification. A persuasive analysis of contingent transformations of political space will require a more sophisticated methodology than that proposed above.

The distinction between constructable and non-constructable transformation is an important and unanticipated result of this research endeavor. I did not anticipate that so many relatively simple transformations could not be generated by continuous linear dynamical systems. This is not simply a
mathematical oddity. It pertains to the fundamental difference between changes that can happen in a gradual evolutionary manner and changes that require a leap, break, or discontinuity of some kind. Social scientists have considered discontinuities in the form of wars, revolutions, economic crises and the like, but the concept of constructability suggests that many seemingly gradual processes actually imply a leap of some sort. The reality of such latent discontinuities has profound implications for social change of all varieties, implications that cannot be addressed within the limits of this paper. Suffice it to say that, once alerted to the possibility of hidden discontinuities, I can locate them in many different contexts.

The analysis of American political space and its transformation made herein is far from definitive. It constitutes an initial stab based upon data that happened to be available. Indeed the initial motivation for this investigation was simply to illustrate the concepts, models, and methodology of political space analysis. Sharper and more rigorous results would be obtained by using data explicitly collected to estimate the transformation of political space. Although the empirical analysis presented above is illustrative, it is certainly not arbitrary. I think that the main conclusions would to be supported by further research using different kinds of information. I am currently collecting information about parties, newspapers, and other politically relevant institutions for purposes of analyzing the transformation of German political space leading up to the Third Reich. Hopefully both the merits and the deficiencies of this article will stimulate new research about political space and its transformation.

REFERENCES


Table 1: Seven Dimensions of American Political Space

1. **Class Ideology**
   - **Left position:**
     - Supports greater economic equality
     - Supports spending on welfare
     - Favors labor unions
   - **Right position:**
     - High confidence in financial institutions and corporations
     - Opposes labor unions
     - Believes success results mainly from hard work
   - *GSS variables:* NATFARE, CONBUS, CONFINAN, CONLABOR, GETAHEAD

2. **Race Ideology**
   - **Left position:**
     - Emphasizes influence of institutional racism
     - Favors action by the state to increase racial equality
     - Encourages interaction between different race and ethnic groups
   - **Right position:**
     - Minimizes influence of institutional racism
     - Opposes state action for racial equality
     - Neutral about interaction between different race and ethnic groups
   - *GSS variables:* NATRACE, RACEMAR

3. **Gender Ideology**
   - **Left position:**
     - Encourages all forms of gender equality
     - Supports the right of abortion
     - Opposes all discrimination against GLBT persons
   - **Right position:**
     - Supports traditional gender relations
     - Opposes most or all forms of abortion
     - Does not extend full civil equality to GLBT persons.
   - *GSS variables:* FEPOL, ABANY, HOMOSEX, FECHLD, FEPRESCH

4. **Civil Liberties Ideology**
   - **Left position:**
     - Supports freedom of speech in all forms
     - Opposes all forms of media censorship
     - Opposes all ideological restrictions on teachers
   - **Right position:**
     - Favors certain decency restrictions on speech
     - Denies media access to inappropriate persons or individuals
     - Favors ideological propriety tests for teachers
   - *GSS variables:* SPKATH, LIBATH, COLATH, SPKRAC, LIBRAC, COLRAC, SPKCOM, LIBCOM, COLCOM, SPKMIL, LIBMIL, COLMIL, SPKHOMO, LIBHOMO, COLHOMO

5. **Crime Ideology**
   - **Left position:**
     - Supports reduction of crime through social reform
     - Favors decriminalization of drugs
     - Opposes capital punishment
   - **Right position:**
     - Favors reduction of crime through strict law enforcement
     - Supports repression of drug traffic
     - Favors capital punishment
   - *GSS variables:* NATCRIME, NATDRUG, NATCITY, CAPPUN, GUNLAW, COURTS
6. **Military Affairs Ideology**
   
   **Left position:**
   - Favors reduction of military establishment
   - Opposes most use of military force in international affairs
   - Favors elimination of nuclear weapons

   **Right position:**
   - Supports robust and energetic military establishment
   - Favors ready use of military force to achieve national interests
   - Favors development of all potentially effective weapons systems

   *GSS variables:* NATARMS, CONARYM

7. **Environmental Ideology**
   
   **Left position:**
   - Perceives widespread and diverse environmental hazards
   - Favors restrictions on economic activities to protect the environment
   - Opposes technologies deemed environmentally harmful

   **Right position:**
   - Does not perceive imminent or dangerous threats to the natural environment
   - Opposes environmentally motivated restrictions on economic activity
   - Favors unrestricted technological development

   *GSS variables:* NATENVIR
**Table 2: Social Aggregates Used to Analyze the Transformation of American Political Space**

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<tbody>
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<td>1.</td>
<td>Liberals</td>
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<td>2.</td>
<td>Politically moderate males</td>
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<td>3.</td>
<td>Politically moderate females</td>
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<td>4.</td>
<td>Conservatives</td>
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<td>5.</td>
<td>Protestant males</td>
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<td>Protestant females</td>
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<td>7.</td>
<td>Catholic males</td>
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<td>8.</td>
<td>Catholic females</td>
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<td>9.</td>
<td>Persons of no religion</td>
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<td>10.</td>
<td>White males</td>
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<td>11.</td>
<td>White females</td>
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<td>12.</td>
<td>Black males</td>
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<td>13.</td>
<td>Black females</td>
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<td>14.</td>
<td>Married males</td>
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<td>15.</td>
<td>Married females</td>
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<td>16.</td>
<td>Never married males</td>
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<td>17.</td>
<td>Never married females</td>
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<td>18.</td>
<td>Persons with less than high school education</td>
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<td>19.</td>
<td>High school only males</td>
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<td>High school only females</td>
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<td>21.</td>
<td>College degree males</td>
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<td>22.</td>
<td>College degree females</td>
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<td>23.</td>
<td>Strong Democrats</td>
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<td>24.</td>
<td>Weak Democrat males</td>
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<td>25.</td>
<td>Weak Democrat females</td>
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<td>26.</td>
<td>Independents</td>
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<td>28.</td>
<td>Weak Republican females</td>
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<td>29.</td>
<td>Strong Republicans</td>
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<td>30.</td>
<td>Family income below average at age 16 males</td>
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<td>31.</td>
<td>Family income below average at age 16 females</td>
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<td>32.</td>
<td>Family income average at age 16 males</td>
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<td>33.</td>
<td>Family income average at age 16 females</td>
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<td>34.</td>
<td>Family income above average at age 16 males</td>
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<td>35.</td>
<td>Family income above average at age 16 females</td>
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<td>36.</td>
<td>Working class males</td>
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<td>37.</td>
<td>Working class females</td>
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<td>38.</td>
<td>Middle class males</td>
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<td>39.</td>
<td>Middle class females</td>
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Table 3: Political Space Transformation Matrices and Transformation Vectors for 1982 to 1993 ($C_1$ and $D_1$), 1993 to 2002 ($C_2$ and $D_2$), and for 1982 to 2002 ($C_3$ and $D_3$)

\[
C_1 = \begin{pmatrix}
0.523 & -0.353 & 0.718 & -0.903 & 0.271 & 0.679 & 0.057 \\
0 & 0.827 & 0.045 & 0.231 & -0.025 & -0.192 & -0.151 \\
0.155 & -0.3 & 0.471 & 0.211 & 0.425 & 0.246 & 0.502 \\
-0.203 & -0.12 & -0.141 & 0.566 & 0.1 & 0.533 & 0.098 \\
-0.553 & -0.092 & 0.059 & -0.636 & 1.363 & 0.365 & 1.229 \\
0.08 & 0.2 & 0.229 & -0.194 & -0.098 & 0.722 & -0.319 \\
0.17 & -0.006 & 0.041 & 0.295 & 0.328 & -0.263 & 0.426 \\
\end{pmatrix}, \quad D_1 = \begin{pmatrix}
3.237 \\
-1.946 \\
-4.314 \\
-5.354 \\
0.552 \\
-2.239 \\
-1.499 \\
\end{pmatrix}
\]

\[
C_2 = \begin{pmatrix}
0.103 & -0.099 & -0.1 & -0.832 & -0.112 & 0.799 & 0.417 \\
-0.021 & 0.539 & -0.287 & -0.042 & 0.126 & 0.09 & 0.588 \\
0.26 & -0.124 & -0.05 & 1.357 & 0.289 & 0.375 & -0.197 \\
-0.083 & 0.045 & -0.114 & 0.938 & 0.084 & 0.043 & -0.054 \\
-0.137 & -0.3 & 0.021 & 0.137 & -0.857 & -0.011 & 0.753 \\
0.236 & 0.065 & -0.224 & 0.239 & -0.286 & 0.424 & 0.891 \\
0.104 & -0.236 & -0.016 & -0.152 & -0.095 & 0.474 & 0.801 \\
\end{pmatrix}, \quad D_2 = \begin{pmatrix}
-1.476 \\
-1.051 \\
-2.925 \\
-2.284 \\
-0.464 \\
10.686 \\
-2.093 \\
\end{pmatrix}
\]

\[
C_3 = \begin{pmatrix}
0.75 & -0.438 & -0.335 & 0.028 & 1.141 & 0.452 & -0.396 \\
-0.126 & 0.634 & -0.286 & 0.251 & 0.506 & -0.15 & 0.166 \\
0.196 & -0.907 & 0.038 & 0.315 & 1.151 & 2.006 & -0.555 \\
-0.155 & -0.135 & -0.005 & 0.428 & 0.126 & 0.518 & -0.202 \\
0.658 & -0.104 & -0.166 & 1.234 & -1.085 & -0.995 & -0.611 \\
0.474 & -0.039 & 0.001 & 0.37 & 0.102 & 0.331 & -0.226 \\
0.315 & -0.111 & 0.392 & -0.234 & 0.089 & 0.307 & -0.11 \\
\end{pmatrix}, \quad D_3 = \begin{pmatrix}
0.977 \\
-1.8 \\
-9.57 \\
-7.146 \\
8.419 \\
-2.624 \\
-2.738 \\
\end{pmatrix}
\]

Order of variables in both rows and columns: class ideology, race ideology, gender ideology, civil liberties ideology, crime ideology, military affairs ideology, environment ideology

<table>
<thead>
<tr>
<th>Determinant[C$_1$] = 0.0042</th>
<th>Determinant[C$_2$] = 0.0054</th>
<th>Determinant[C$_3$] = 0.0023</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalues[C$_1$] =</td>
<td>Eigenvalues[C$_2$] =</td>
<td>Eigenvalues[C$_3$] =</td>
</tr>
<tr>
<td>1.493</td>
<td>1.326</td>
<td>-1.748</td>
</tr>
<tr>
<td>1.348</td>
<td>-0.829</td>
<td>0.663 + 0.641i</td>
</tr>
<tr>
<td>0.855</td>
<td>0.807</td>
<td>0.663 - 0.641i</td>
</tr>
<tr>
<td>0.487 - 0.362i</td>
<td>0.419 - 0.307i</td>
<td>0.752</td>
</tr>
<tr>
<td>0.487 - 0.362i</td>
<td>0.419 - 0.307i</td>
<td>0.502</td>
</tr>
<tr>
<td>0.196</td>
<td>-0.315</td>
<td>0.17</td>
</tr>
<tr>
<td>0.034</td>
<td>0.072</td>
<td>-0.024</td>
</tr>
</tbody>
</table>
Table 5: Infinitesimal Generator ($A_i$) and Infinitesimal Vector ($B_i$) for the 1982 to 1993 Political Space Transformation

$$A_i = \begin{pmatrix}
-0.074 & -0.051 & 0.108 & -0.125 & 0.027 & 0.107 & -0.043 \\
0.029 & 0.017 & 0.012 & 0.057 & -0.004 & -0.087 & -0.069 \\
0.042 & -0.03 & -0.091 & 0.069 & 0.029 & 0.000 & 0.079 \\
-0.039 & -0.047 & -0.023 & -0.058 & 0.016 & 0.107 & 0.046 \\
-0.204 & -0.152 & 0.04 & -0.259 & 0.031 & 0.337 & 0.292 \\
0.02 & 0.046 & 0.03 & -0.004 & -0.006 & -0.068 & -0.08 \\
0.163 & 0.133 & -0.017 & 0.212 & 0.023 & -0.305 & -0.255 \\
\end{pmatrix}$$

$$B_i = \begin{pmatrix}
0.083 \\
-0.038 \\
-0.025 \\
0.038 \\
0.289 \\
-0.086 \\
-0.293 \\
\end{pmatrix}$$
Table 6: Eigenvalues and Dominant Eigenvectors of Infinitesimal Generator $A_t$, and Equilibrium Position for the 1982 to 1993 Political Space Transformation

Eigenvalues of $A_t = \begin{pmatrix} -0.309 \\ -0.148 \\ -0.045 - 0.058i \\ -0.045 - 0.058i \\ 0.036 \\ 0.027 \\ -0.014 \end{pmatrix}$

Dominant Eigenvector of $A_t = \begin{pmatrix} 0.635 \\ -0.086 \\ 0.451 \\ -0.117 \\ 0.552 \\ -0.012 \\ 0.259 \end{pmatrix}$, Sub-Dominant Eigenvector of $A_t = \begin{pmatrix} 0.776 \\ -0.129 \\ 0.441 \\ -0.162 \\ 0.333 \\ 0.0442 \\ 0.218 \end{pmatrix}$

Equilibrium Position of 1982 to 1993 Transformation = $\begin{pmatrix} 0.576 \\ 0.452 \\ -1.16 \\ -1.21 \\ 0.92 \\ -0.805 \\ -0.436 \end{pmatrix}$
Table 7: Constructable Variation of 1993 to 2002 Political Transformation Matrix (VC$_2$), Eigenvalues of VC$_2$, Infinitesimal Vector for VC$_2$ (VB$_2$), Infinitesimal Generator of VC$_2$ (AC$_2$), Eigenvalues of AC$_2$, and Dominant Eigenvector of AC$_2$.

\[
\begin{bmatrix}
-0.103 & -0.099 & -0.1 & -0.832 & 0.112 & 0.799 & 0.417 \\
0.021 & 0.539 & -0.287 & -0.042 & -0.126 & 0.09 & 0.588 \\
-0.26 & -0.124 & -0.05 & 1.357 & -0.289 & 0.375 & -0.197 \\
0.083 & 0.045 & -0.114 & 0.938 & -0.084 & 0.043 & -0.054 \\
0.137 & -0.3 & 0.021 & 0.137 & 0.857 & -0.011 & 0.753 \\
-0.236 & 0.065 & -0.224 & 0.239 & 0.286 & 0.424 & 0.891 \\
-0.104 & -0.236 & -0.016 & -0.152 & 0.095 & 0.474 & 0.801 \\
\end{bmatrix}
\]

\[
VC_2 = \begin{bmatrix}
1.288 \\
0.776 - 0.305i \\
0.776 - 0.305i \\
0.538 \\
-0.017 + 0.426i \\
-0.017 - 0.426i \\
0.062 \\
\end{bmatrix}
\]

Eigenvalues of VC$_2$ = \[
\begin{bmatrix}
-4.189 \\
-0.68 \\
-4.825 \\
-0.089 \\
1.297 \\
-1.087 \\
-0.854 \\
\end{bmatrix}
\]

\[
VB_2 = \begin{bmatrix}
-0.146 & -0.073 & -0.014 & -0.19 & -0.05 & 0.268 & -0.029 \\
-0.022 & -0.113 & -0.131 & 0.115 & -0.073 & 0.069 & 0.107 \\
-0.081 & -0.193 & -0.295 & 0.346 & -0.167 & 0.313 & -0.06 \\
0.014 & -0.007 & -0.031 & 0.028 & -0.02 & 0.013 & -0.007 \\
0.005 & -0.024 & -0.022 & 0.062 & -0.019 & -0.05 & 0.143 \\
-0.108 & -0.008 & -0.087 & 0.05 & 0.005 & 0.015 & 0.11 \\
0.015 & -0.051 & -0.001 & -0.019 & -0.006 & 0.082 & -0.051 \\
\end{bmatrix}
\]

\[
AC_2 = \begin{bmatrix}
-0.309 \\
-0.095 - 0.179i \\
-0.095 - 0.179i \\
-0.069 \\
-0.02 - 0.042i \\
-0.02 - 0.042i \\
0.028 \\
\end{bmatrix}
\]

Eigenvalues of AC$_2$ = \[
\begin{bmatrix}
0.416 \\
0.305 \\
0.097 \\
0.041 \\
0.562 \\
0.521 \\
0.37 \\
\end{bmatrix}
\]

Dominant Eigenvector of AC$_2$ = \[
\begin{bmatrix}
0.416 \\
0.305 \\
0.097 \\
0.041 \\
0.562 \\
0.521 \\
0.37 \\
\end{bmatrix}
\]
Figure 1: Elementary Examples of Constructable Translations

1. Asymmetric Stretching

2. Counter-Clockwise Rotation

3. Diagonal Dilation

4. Translation
Figure 2: Schematic Representations of Two-Dimensional Constructable and Non-Constructable Transformations of Political Space

Constructable Transformations

1. $180^\circ$ Rotation
2. Dilation and Contraction

Non-Constructable Transformations

3. Reflection About Diagonal
4. Reflection About Vertical Axis
Figure 3: Projected Evolution of Center Position over 11 and over 100 Years using Infinitesimal Generator $A_1$ and Infinitesimal Vector $B_1$.
Figure: 4: Distance and Angle Relations between Four Different Political Positions (Left, Right, Center, and Mixed) Over the 1982 to 1993 Political Space Transformation
Figure 5: Shadow Destiny of 1982 to 1993 Political Space Transformation – Evolution of Shadow Political Locations and of Shadow Angles with Other Political Positions
Figure 6: Projected Evolution of Right and Mixed Positions between 1993 and 2002

<table>
<thead>
<tr>
<th>Class</th>
<th>Race</th>
<th>Gender</th>
<th>Civil Liberties</th>
<th>Crime</th>
<th>Military Affairs</th>
<th>Environment</th>
</tr>
</thead>
</table>
Figure 7: Imputed Long Term Consequences of 9/11 Illustrated by 50 Year Projections of the Center Position

<table>
<thead>
<tr>
<th>Class</th>
<th>Race</th>
<th>Gender</th>
<th>Civil Liberties</th>
<th>Crime</th>
<th>Military Affairs</th>
<th>Environment</th>
</tr>
</thead>
</table>

Location on Dimension

With 9/11

Year after 1993

Location on Dimension

Without 9/11

Year after 1993